MHT CET 2023 : 10th May Evening Shift

Mathematics

Question 1

The value of c for the function $f(x) = \log x$ on [1, e] if LMVT can be applied, is

Options:

A. $e - 2$

B. $e + 1$

C. $e - 1$

D. e

Answer: C

Solution:

$$f(x) = \log x$$
$$\Rightarrow f'(x) = \frac{1}{x}$$

By Lagrange's Mean value theorem,

$$f'(c) = \frac{f(e) - f(1)}{e - 1}$$
$$\Rightarrow \frac{1}{c} = \frac{\log e - \log 1}{e - 1}$$
$$\Rightarrow \frac{1}{c} = \frac{1}{e - 1}$$
$$\Rightarrow c = e - 1$$

Question 2

The three ships namely A, B and C sail from India to Africa. If the odds in favour of the ships reaching safely are $2 : 5, 3 : 7$ and $6 : 11$ respectively,

then probability of all of them arriving safely is

Options:

A. $\frac{18}{595}$

B. $\frac{11}{34}$

C. $\frac{196}{217}$

D. $\frac{1}{595}$

Answer: A

Solution:

The probability that ship 'A' reaches safely is $\mathrm{P(A)} = \frac{2}{2+5} = \frac{2}{7}$

The probability that ship 'B' reaches safely is $\mathrm{P(B)} = \frac{3}{3+7} = \frac{3}{10}$

The probability that ship 'C' reaches safely is $\mathrm{P(C)} = \frac{6}{6+11} = \frac{6}{17}$

$\therefore$ Probability that all of them arriving safely

$$= P(A \cap B \cap C)$$
$$= P(A) \cdot P(B) \cdot P(C)$$

[Since A, B, C are all independent events]

$$= \frac{2}{7} \times \frac{3}{10} \times \frac{6}{17}$$
$$= \frac{18}{595}$$

Question 3

If p and q are true statements and r and s are false statements, then the truth values of the statement patterns $(p \wedge q) \vee r$ and $(\mathrm{p} \vee \mathrm{s}) \leftrightarrow (\mathrm{q} \wedge \mathrm{r})$ are respectively

Options:

A. F, T

B. T, T

C. F, F

D. T, F

Answer: D

Solution:

$$\begin{array}{ll} (p \wedge q) \vee r & (p \vee s) \leftrightarrow (q \wedge r) \\ \equiv (T \wedge T) \vee F & \equiv (T \vee F) \leftrightarrow (T \wedge F) \\ \equiv T \vee F & \equiv T \leftrightarrow F \\ \equiv T & \equiv F \end{array}$$

Question 4

If the sum of mean and variance of a Binomial Distribution is $\frac{15}{2}$ for 10 trials, then the variance is

Options:

A. 1.5

B. 2.5

C. 4.5

D. 3.5

Answer: B

Solution:

$$\text{Mean} + \text{variance} = \frac{15}{2}$$
$$\Rightarrow np + npq = \frac{15}{2}$$
$$\Rightarrow np + np(1-p) = \frac{15}{2} \quad \ldots [\because p + q = 1]$$
$$\Rightarrow n\left(2p - p^2\right) = \frac{15}{2}$$
$$\Rightarrow 2p - p^2 = \frac{15}{2 \times 10}$$
$$\Rightarrow 4p^2 - 8p + 3 = 0$$
$$\Rightarrow (2p-3)(2p-1) = 0$$
$$\Rightarrow p = \frac{3}{2} \text{ or } p = \frac{1}{2}$$
$$\therefore \quad p = \frac{1}{2} \quad \ldots [\because 0 < p < 1]$$
$$\therefore \quad q = 1 - \frac{1}{2} = \frac{1}{2}$$
$$\text{Variance} = npq = 10 \times \frac{1}{2} \times \frac{1}{2} = 2.5$$

Question 5

If $(\bar{a} \times \bar{b}) \times \bar{c} = -5\bar{a} + 4\bar{b}$ and $\bar{a} \cdot \bar{b} = 3$, then the value of $\bar{a} \times (\bar{b} \times \bar{c})$ is

Options:

A. $3\bar{b} - 4\bar{c}$

B. $4\bar{a} - 3\bar{b}$

C. $4\bar{b} - 3\bar{c}$

D. $3\bar{a} - 4\bar{c}$

Answer: C

Solution:

$$(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$$
$$\text{But, } (\bar{a} \times \bar{b}) \times \bar{c} = -5\bar{a} + 4\bar{b}$$
$$\therefore \quad -5\bar{a} + 4\bar{b} = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{b} \cdot \bar{c})\bar{a}$$

Comparing, we get

$\overline{a} \cdot \overline{c} = 4$

$\therefore \quad \overline{a} \times (\overline{b} \times \overline{c}) = (\overline{a} \cdot \overline{c})\overline{b} - (\overline{a} \cdot \overline{b})\overline{c}$

$= 4\overline{b} - 3\overline{c} \quad \ldots [\overline{a} \cdot \overline{b} = 3 \text{ (given)}]$

Question 6

The plane through the intersection of planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to Y-axis also passes through the point

Options:

A. $(3, 3, -1)$

B. $(-3, 0, 1)$

C. $(3, 2, 1)$

D. $(-3, 0, -1)$

Answer: C

Solution:

Equation of plane through the intersection of given planes is

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0 \quad \ldots \text{(i)}$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + 4\lambda - 1 = 0$

Since the plane is parallel to Y-axis.

$\therefore \quad 1 + 3\lambda = 0$

$\Rightarrow \lambda = \frac{-1}{3}$

Substituting $\lambda = \frac{-1}{3}$ in (i), we get

$(x + y + z - 1) - \frac{1}{3}(2x + 3y - z + 4) = 0$

$\Rightarrow x + 4z - 7 = 0$

Point $(3, 2, 1)$ satisfies this equation.

Question 7

Let $\mathrm{f}(x)=\mathrm{e}^{x}-x$ and $\mathrm{g}(x)=x^{2}-x, \forall x \in \mathrm{R}$, then the set of all $x \in \mathrm{R}$, where the function $\mathrm{h}(x)=(\mathrm{fog})(x)$ is increasing is

Options:

A. $\left[0, \frac{1}{2}\right] \cup[1, \infty)$

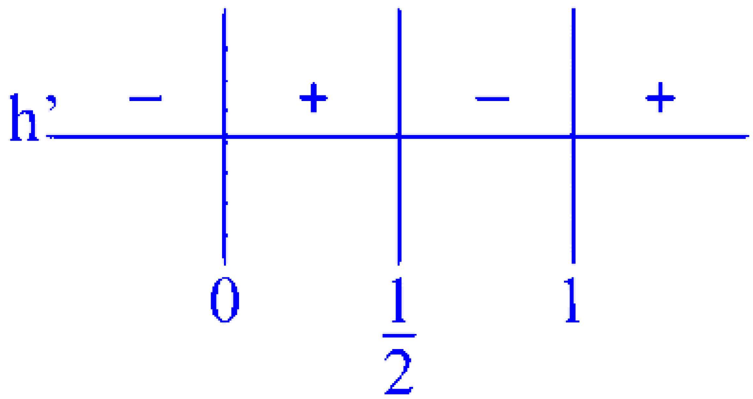
B. $\left[-1,-\frac{1}{2}\right] \cup\left[\frac{1}{2}, \infty\right)$

C. $[0, \infty)$

D. $\left[-\frac{1}{2}, 0\right] \cup[1, \infty)$

Answer: A

Solution:



$$
\begin{aligned}
& \mathrm{h}(x)=(f \circ g)(x) \\
& \Rightarrow \mathrm{h}(x)=\mathrm{f}\left(x^{2}-x\right) \\
& \mathrm{h}(x)=\mathrm{e}^{x^{2}-x}-x^{2}+x \\
\therefore\ & \mathrm{h}'(x)=\mathrm{e}^{x^{2}-x}(2x-1)-2x+1 \\
& \Rightarrow \mathrm{h}'(x)=\left(\mathrm{e}^{x^{2}-x}-1\right)(2x-1)
\end{aligned}
$$

For function $\mathrm{h}(x)$ to be increasing,

$\mathrm{h}'(x) \geq 0$

$\Rightarrow\left(\mathrm{e}^{x^{2}-x}-1\right)(2x-1) \geq 0$

$\Rightarrow x \in\left[0, \frac{1}{2}\right] \cup[1, \infty)$

Question 8

Let f be a differentiable function such that $\mathrm{f}(1) = 2$ and $\mathrm{f}'(x) = \mathrm{f}(x)$, for all $x \in \mathrm{R}$. If $\mathrm{h}(x) = \mathrm{f}(\mathrm{f}(x))$, then $\mathrm{h}'(1)$ is equal to

Options:

A. $4\mathrm{e}^2$

B. $4\mathrm{e}$

C. $2\mathrm{e}$

D. $2\mathrm{e}^2$

Answer: B

Solution:

Given: $\mathrm{f}'(x) = \mathrm{f}(x)$ for all $x \in \mathrm{R}$

$\Rightarrow \frac{\mathrm{f}'(x)}{\mathrm{f}(x)} = 1$

Integrating on both sides, we get

$$\log|\mathrm{f}(x)| = x + \mathrm{c}$$
$$\Rightarrow \mathrm{f}(x) = \mathrm{e}^{x+\mathrm{c}}$$
$$\Rightarrow \mathrm{f}(x) = \mathrm{e}^{x} \cdot \mathrm{e}^{\mathrm{c}}$$
$$\Rightarrow \mathrm{f}(x) = \mathrm{e}^{x} \cdot \mathrm{c}_1 \quad \ldots \text{(i)} \ [\mathrm{e}^{\mathrm{c}} = \mathrm{c}_1]$$

As $\mathrm{f}(1) = 2$

$$\therefore \quad c_1 \cdot e = 2$$
$$\Rightarrow c_1 = \frac{2}{e}$$

Equation (i) becomes

$f(x) = \mathrm{e}^{x} \cdot \frac{2}{\mathrm{e}}$

Now, $\mathrm{h}(x) = \mathrm{f}(\mathrm{f}(x))$

$$\therefore \quad h'(x) = f'(f(x)) \times f'(x)$$
$$\therefore \quad h'(1) = f'(f(1)) \times f'(1)$$
$$\Rightarrow h'(1) = f'(2) \times f'(1)$$
$$\Rightarrow h'(1) = e^2 \times \frac{2}{e} \times 2$$
$$\Rightarrow h'(1) = 4e$$

Question 9

The joint equation of a pair of lines passing through the origin and making an angle of $\frac{\pi}{4}$ with the line $3x + 2y - 8 = 0$ is

Options:

A. $5x^2 + 24xy - 5y^2 = 0$

B. $5x^2 - 24xy + 5y^2 = 0$

C. $5x^2 - 24xy - 5y^2 = 0$

D. $5x^2 + 24xy + 5y^2 = 0$

Answer: A

Solution:

The slope of line $3x + 2y - 8 = 0$ is $m_1 = \frac{-3}{2}$

Let m be the slope of one of the lines making angle $\frac{\pi}{4}$ with $3x + 2y - 8 = 0$

$$\therefore \quad \tan\frac{\pi}{4} = \left|\frac{m - m_1}{1 + m_1}\right|$$
$$\Rightarrow 1 = \left|\frac{m - \left(\frac{-3}{2}\right)}{1 + m\left(\frac{-3}{2}\right)}\right|$$
$$\Rightarrow 1 = \left|\frac{2m + 3}{2 - 3m}\right|$$

Squaring on both sides, we get

$$(2 - 3m)^2 = (2m + 3)^2$$
$$\Rightarrow 5m^2 - 24m - 5 = 0$$

This is the auxiliary equation of two lines and their joint equation is obtained by putting

$\mathrm{m} = \frac{y}{x}$

The joint equation of the lines is

$$5\left(\frac{y}{x}\right)^2 - 24\left(\frac{y}{x}\right) - 5 = 0$$
$$\text{i.e., } 5x^2 + 24xy - 5y^2 = 0$$

Question 10

Two sides of a square are along the lines $5x - 12y + 39 = 0$ and $5x - 12y + 78 = 0$, then area of the square is

Options:

A. 9 sq. units.

B. $\frac{1}{3}$ sq. units.

C. 18 sq. units.

D. 3 sq. units.

Answer: A

Solution:

Given equations of lines are $5x-12y+39=0$ and $5x-12y+78=0$.

Slope of $5x-12y+39=0$ is $\frac{5}{12}$

Slope of $5x-12y+78=0$ is $\frac{5}{12}$

$\therefore$ Lines are parallel.

$\therefore$ Distance between two parallel lines $=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

$$=\left|\frac{39-78}{\sqrt{5^2+(-12)^2}}\right|$$
$=3$ units

$\therefore$ Side of the square $=3$ units

$\therefore$ Area of square $=3^2=9$ sq. units

Question 11

The value of $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$ is

Options:

A. $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$, where c is a constant of integration.

B. $(x^4+1)^{\frac{1}{4}}+c$, where c is a constant of integration.

C. $-(x^4+1)^{\frac{1}{4}}+c$, where c is a constant of integration.

D. $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+$ c, where c is a constant of integration.

Answer: D

Solution:

Let ,I $=\int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$ d$x=\int \frac{\mathrm{d}x}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}$

Put $1+\frac{1}{x^4}=\mathrm{t} \Rightarrow \frac{-4}{x^5}\,\mathrm{d}x=\mathrm{dt}$

$$\therefore=-\frac{1}{4}\int \frac{dt}{t^{\frac{3}{4}}}$$
$$=-\frac{1}{4}\times 4t^{\frac{1}{4}}+c=-t^{\frac{1}{4}}+c$$
$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+c=-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$$

Question 12

$\int \frac{5\tan x}{\tan x-2}\,\mathrm{d}x=x+\mathrm{a}\log|\sin x-2\cos x|+\mathrm{c}$, (where c is a constant of integration), then the value of a is

Options:

A. 1

B. $\frac{1}{2}$

C. 2

D. 3

Answer: C

Solution:

$$\text{Let}\quad \mathrm{I}=\int \frac{5\tan x}{\tan x-2}\,\mathrm{d}x$$
$$=\int \frac{5\sin x}{\sin x-2\cos x}\,\mathrm{d}x$$

Let $5\sin x=\mathrm{A}(\sin x-2\cos x)+\mathrm{B}\cdot\frac{\mathrm{d}}{\mathrm{d}x}(\sin x-2\cos x)$

$\therefore \quad 5\sin x = \mathrm{A}(\sin x - 2\cos x) + \mathrm{B}(\cos x + 2\sin x)$
$\therefore \quad \mathrm{A} + 2\,\mathrm{B} = 5$ and $-2\,\mathrm{A} + \mathrm{B} = 0$

Solving these equations, we get

$$\mathrm{A} = 1,\ \mathrm{B} = 2$$

$$\therefore \quad \mathrm{I} = \int 1\, dx + 2\int \frac{\cos x + 2\sin x}{\sin x - 2\cos x} dx$$
$$= x + 2\log|\sin x - 2\cos x| + c$$

But $\int \frac{5\tan x}{\tan x - 2} dx = x + a\log|\sin x - 2\cos x| + c$

Comparing, we get $a = 2$

Question 13

Let $S = \{t \in R / f(x) = |x - \pi|\left(e^{|x|} - 1\right)\sin|x|$ is not differentiable at $t\}$, then S is

Options:

A. ϕ (an empty set)

B. $\{0\}$

C. $\{\pi\}$

D. $\{0, \pi\}$

Answer: A

Solution:

Differentiability at $x = \pi$:

L.h.lim

$$= \lim_{h \to 0} \frac{|\pi - h - \pi|\left(e^{|\pi - h|} - 1\right)\sin|\pi - h| - 0}{-h} = 0$$

R.h.lim

$$= \lim_{h \to 0} \frac{|\pi + h - \pi|\left(e^{|\pi + h|} - 1\right)\sin|\pi + h| - 0}{h} = 0$$

Differentiability at $x = \mathbf{0}$:

$$\text{L.h.lim} = \lim_{h \to 0} \frac{|-h - \pi|\left(e^{|-h|} - 1\right)\sin|-h| - 0}{-h} = 0$$

$\text{R.h.lim} = \lim_{h \to 0} \frac{|h-\pi|\left(e^{|h|}-1\right) \sin |h|-0}{h} = 0$

The function $f(x)$ is differentiable at $x = 0, \pi$.

$\Rightarrow$ Set S is an empty set.

Question 14

The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to

Options:

A. $\frac{\sqrt{17}}{2} + 1$

B. $\frac{\sqrt{17}-1}{2}$

C. $\frac{1+\sqrt{17}}{2}$

D. $\frac{\sqrt{17}}{2} - 1$

Answer: C

Solution:

$f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$

$f(x)$ is defined, if

$$-1 \leq \frac{|x|+5}{x^2+1} \leq 1$$
$$\Rightarrow 0 \leq \frac{|x|+5}{x^2+1} \leq 1$$
$$\Rightarrow x^2-|x|-4 \geq 0$$
$$\Rightarrow \left(|x|-\frac{1-\sqrt{17}}{2}\right)\left(|x|-\frac{1+\sqrt{17}}{2}\right) \geq 0$$
$$\Rightarrow |x| \geq \frac{1+\sqrt{17}}{2}$$
or $|x| \leq \frac{1-\sqrt{17}}{2}$, which is not possible
$$\Rightarrow x \in \left(-\infty,-\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$$
$$\Rightarrow \mathrm{a}=\frac{1+\sqrt{17}}{2}$$

Question 15

The perpendicular distance of the origin from the plane $x-3y+4z-6=0$ is

Options:

A. 6

B. $\frac{6}{\sqrt{26}}$

C. $\frac{1}{\sqrt{26}}$

D. $\frac{3}{\sqrt{26}}$

Answer: B

Solution:

Length of perpendicular from point $O(0,0,0)$ to plane $x-3y+4z-6=0$ is given by

$$d=\left|\frac{0(1)+0(-3)+0(4)-6}{\sqrt{(1)^2+(-3)^2+(4)^2}}\right|$$
$$=\left|\frac{-6}{\sqrt{1+9+16}}\right|=\frac{6}{\sqrt{26}}$$

Question 16

The area bounded by the X-axis and the curve $y = x(x-2)(x+1)$ is

Options:

A. $\frac{37}{12}$ sq. units

B. $\frac{27}{12}$ sq. units

C. $\frac{37}{4}$ sq. units

D. $\frac{27}{13}$ sq. units

Answer: A

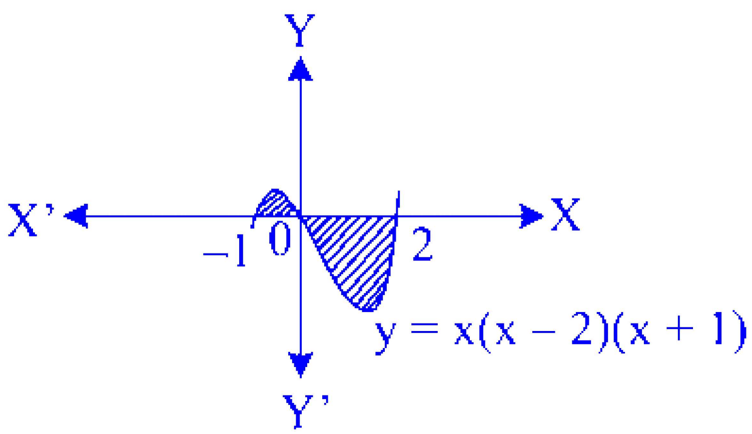
Solution:

For X-axis,

$y = 0$

$\therefore \quad x(x-2)(x+1) = 0$

$\Rightarrow x = 0$ or $x = 2$ or $x = -1$



Required area $= \int_{-1}^{0} y\,dx + \left|\int_{0}^{2} y\,dx\right|$

$$= \int_{-1}^{0} x(x-2)(x+1)\mathrm{d}x + \left|\int_{0}^{2} x(x-2)(x+1)\mathrm{d}x\right|$$

$$= \int_{-1}^{0} \left(x^3 - x^2 - 2x\right)\mathrm{d}x + \left|\int_{0}^{2} \left(x^3 - x^2 - 2x\right)\mathrm{d}x\right|$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0} + \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{0}^{2}\right|$$

$$= \frac{5}{12} + \left|-\frac{8}{3}\right|$$

$$= \frac{37}{12} \text{ sq. units}$$

Question 17

Let $\mathrm{f} : \mathrm{R} \rightarrow \mathrm{R}$ and $\mathrm{g} : \mathrm{R} \rightarrow \mathrm{R}$ be continuous functions. Then the value of the integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} [\mathrm{f}(x) + \mathrm{f}(-x)][\mathrm{g}(x) - \mathrm{g}(-x)]\mathrm{d}x$ is

Options:

A. π

B. 1

C. -1

D. 0

Answer: D

Solution:

Let $\mathrm{h}(x) = [\mathrm{f}(x) + \mathrm{f}(-x)][\mathrm{g}(x) - \mathrm{g}(-x)]$

$$\begin{aligned} \therefore \quad \mathrm{h}(-x) &= [\mathrm{f}(-x) + \mathrm{f}(x)][\mathrm{g}(-x) - \mathrm{g}(x)] \\ &= -[\mathrm{f}(x) + \mathrm{f}(-x)][\mathrm{g}(x) - \mathrm{g}(-x)] \\ &= -\mathrm{h}(x) \end{aligned}$$

$\therefore \quad \mathrm{h}(x)$ is an odd function.

$$\therefore \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \mathrm{h}(x) = 0$$

Question 18

If $\bar{p} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{q} = \hat{i} - 2\hat{j} + \hat{k}$. Then a vector of magnitude $5\sqrt{3}$ units perpendicular to the vector $\bar{q}$ and coplanar with $\bar{p}$ and $\bar{q}$ is

Options:

A. $5(\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}})$

B. $5(\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}})$

C. $5(\hat{\mathrm{i}} - \hat{\mathrm{j}} - \hat{\mathrm{k}})$

D. $5(\hat{i} + \hat{j} + \hat{k})$

Answer: D

Solution:

Let $\bar{r} = a\hat{i} + b\hat{j} + c\hat{k}$

As $\overline{\mathrm{r}}$ is perpendicular to $\overline{\mathrm{q}}$.

$$\therefore \quad \bar{r} \cdot \bar{q} = 0$$
$$\Rightarrow a - 2b + c = 0 \quad \text{.... (i)}$$

Also, $\bar{r}$ is coplanar with vectors $\bar{p}$ and $\bar{q}$

$$\therefore \quad [\bar{p} \quad \bar{q} \quad \bar{r}] = 0$$
$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ a & b & c \end{vmatrix} = 0$$
$$\Rightarrow 3a - 3c = 0$$
$$\Rightarrow a - c = 0$$
$$\Rightarrow a = c \quad \text{..... (ii)}$$

From (i) and (ii), we get

$$\mathrm{b} = \mathrm{c}$$
$$\therefore \quad \overline{\mathrm{r}} = \hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}$$

Now, the magnitude of required vector is $5\sqrt{3}$ units.

$\therefore$ Required vector $= 5\sqrt{3} \times \frac{\overline{r}}{|\overrightarrow{r}|}$

$$= 5\sqrt{3} \times \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} = 5(\hat{i} + \hat{j} + \hat{k})$$

Question 19

The value of $\frac{{}^{10}C_r}{{}^{11}C_r}$, when both the numerator and denominator are at their greatest values, is

Options:

A. $\frac{6}{11}$

B. $\frac{1}{11}$

C. $\frac{4}{11}$

D. $\frac{3}{11}$

Answer: A

Solution:

Greatest value of ${}^{10}C_r$ is at $r = 5$

Greatest value of ${}^{11}C_r$ is at $r = 5$

$\therefore \quad \frac{{}^{10}C_r}{{}^{11}C_r} = \frac{{}^{10}C_5}{{}^{11}C_5} = \frac{\frac{10!}{5!5!}}{\frac{11!}{5!6!}} = \frac{6}{11}$

Question 20

The general solution of the differential equation $\frac{dy}{dx} + \left(\frac{3x^2}{1+x^3}\right)y = \frac{1}{x^3+1}$ is

Options:

A. $y\left(1 + x^3\right) = x^3 + c$, where c is a constant of integration.

B. $y\left(1 + x^3\right) = x + c$, where c is a constant of integration.

C. $y\left(1+x^3\right)=x^2+c$, where c is a constant of integration.

D. $y\left(1+x^3\right)=2x+c$, where c is a constant of integration.

Answer: B

Solution:

Given differential equation is

$$\frac{dy}{dx}+\left(\frac{3x^2}{1+x^3}\right)y=\frac{1}{x^3+1}$$

Here, $P=\frac{3x^2}{1+x^3}, Q=\frac{1}{x^3+1}$

$\therefore \quad \text{I.F.} =e^{\int\frac{3x^2}{1+x^3}\,dx}=e^{\log\left(1+x^3\right)}=\left(1+x^3\right)$

$\therefore$ Solution of the given equation is

$$y\left(1+x^3\right)=\int\frac{1}{1+x^3}\cdot\left(1+x^3\right)dx+c$$

$$\Rightarrow y\left(1+x^3\right)=x+c$$

Question 21

If y is a function of x and $\log(x+y)=2xy$, then $\frac{dy}{dx}$ at $x=0$ is

Options:

A. 0

B. -1

C. 1

D. 2

Answer: C

Solution:

$\log(x+y)=2xy$ ...... (i)

Differentiating both sides w.r.t. x, we get

$$\left(\frac{1}{x+y}\right)\left(1+\frac{dy}{dx}\right)=2\left(x\frac{dy}{dx}+y\right)$$
$$\Rightarrow \frac{dy}{dx}=\frac{1-2xy-2y^2}{2x^2+2xy-1}$$
Putting $x=0$ in (i), we get
$y=1$
$$\therefore \left(\frac{dy}{dx}\right)_{x=0}=\frac{1-0-2}{0+0-1}=1$$

Question 22

The displacement 'S' of a moving particle at a time t is given by $S=5+\frac{48}{t}+t^3$. Then its acceleration when the velocity is zero, is

Options:

A. 12

B. 20

C. 16

D. 24

Answer: D

Solution:

Given, $S=5+\frac{48}{t}+t^3$

velocity(V) $=\frac{dS}{dt}=0-\frac{48}{t^2}+3t^2$

$V=\frac{-48}{t^2}+3t^2$ ..... (i)

But V $=0$ ...[Given]

$$\Rightarrow \frac{-48}{t^2}+3t^2=0$$
$$\Rightarrow t=2$$

Now,

$$
\begin{aligned}
A &= \frac{dV}{dt} \\
&= \frac{d}{dt}\left(\frac{-48}{t^2} + 3t^2\right) \\
&= \frac{96}{t^3} + 6t
\end{aligned}
$$

Acceleration at t $= 2$ is $\frac{96}{8} + 12 = 24$

Question 23

If $\bar{a}$ and $\bar{b}$ are two unit vectors such that $\bar{a} + 2\bar{b}$ and $5\bar{a} - 4\bar{b}$ are perpendicular to each other, then the angle between $\bar{a}$ and $\bar{b}$ is

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

Answer: A

Solution:

Given that, $\overline{\mathrm{a}} + 2\overline{\mathrm{b}}$ and $5\overline{\mathrm{a}} - 4\overline{\mathrm{b}}$ are perpendicular to each other.

$$
\begin{aligned}
&\therefore \quad (\bar{a} + 2\bar{b}) \cdot (5\bar{a} - 4\bar{b}) = 0 \\
&\Rightarrow 5|\bar{a}|^2 - 8|\bar{b}|^2 - 4\bar{a} \cdot \bar{b} + 10\bar{b} \cdot \bar{a} = 0 \\
&\Rightarrow -3 + 6\bar{a} \cdot \bar{b} = 0 \quad \ldots [|\bar{a}| = |\bar{b}| = 1] \\
&\Rightarrow 6|\bar{a}||\bar{b}| \cos\theta = 3 \\
&\Rightarrow \cos\theta = \frac{1}{2} \\
&\Rightarrow \theta = \frac{\pi}{3}
\end{aligned}
$$

Question 24

The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is

Options:

A. $\frac{5\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer: D

Solution:

$$\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$$
$$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$
$$=\frac{3\pi}{4}$$

Question 25

$$\int_0^\pi \frac{x\tan x}{\sec x+\cos x}dx=$$

Options:

A. $\frac{\pi}{8}$

B. $-\frac{\pi^2}{8}$

C. $\frac{\pi^2}{4}$

D. $-\frac{\pi^2}{4}$

Answer: C

Solution:

$$\text{Let I} = \int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx \quad \ldots \text{(i)}$$

$$I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \cos x} dx \quad \ldots\ldots \text{(ii)}$$

$$\ldots \left[\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right]$$

Adding (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \cos x} dx$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

Put $\cos x = t \Rightarrow \sin x \, dx = -dt$

$$\therefore \quad I = -\frac{\pi}{2} \int_1^{-1} \frac{dt}{1 + t^2}$$

$$= -\frac{\pi}{2}\left[\tan^{-1} t\right]_1^{-1}$$

$$= \left(-\frac{\pi}{2}\right)\left(-\frac{\pi}{2}\right) = \frac{\pi^2}{4}$$

Question 26

If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, then which of the following statement is true?

Options:

A. $a + b - c = abc$

B. $a + b + c = 2abc$

C. $abc = 1$

D. $a + b + c = abc$

Answer: D

Solution:

$$\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$$
$$\Rightarrow \tan^{-1}\left(\frac{a+b+c-abc}{1-ab-bc-ca}\right) = \pi$$
$$\Rightarrow \frac{a+b+c-abc}{1-ab-bc-ca} = \tan\pi = 0$$
$$\Rightarrow a+b+c-abc = 0$$
$$\Rightarrow a+b+c = abc$$

Question 27

If $\mathrm{f}(x) = \frac{4}{x^4}\left[1 - \cos\frac{x}{2} - \cos\frac{x}{4} + \cos\frac{x}{2}\cdot\cos\frac{x}{4}\right]$ is continuous at $x = 0$, then $\mathrm{f}(0)$ is

Options:

A. $\frac{1}{32}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{1}{64}$

Answer: D

Solution:

$$\begin{aligned} f(x) &= \frac{4}{x^4}\left[1 - \cos\frac{x}{2} - \cos\frac{x}{4} + \cos\frac{x}{2}\cdot\cos\frac{x}{4}\right] \\ &= \frac{4}{x^4}\left[\left(1-\cos\frac{x}{2}\right) - \cos\frac{x}{4}\left(1-\cos\frac{x}{2}\right)\right] \\ &= \frac{4}{x^4}\left[\left(1-\cos\frac{x}{2}\right)\left(1-\cos\frac{x}{4}\right)\right] \end{aligned}$$

$\mathrm{f}(x)$ is continuous at $x = 0$. .... [Given]

$\therefore \quad \mathrm{f}(0)=\lim_{x\to 0}\mathrm{f}(x)$

$=\lim_{x\to 0}\frac{4}{x^4}\left(1-\cos\frac{x}{2}\right)\left(1-\cos\frac{x}{4}\right)$

$=4\lim_{x\to 0}\left(\frac{2\sin^2\frac{x}{4}}{x^2}\right)\times\left(\frac{2\sin^2\frac{x}{8}}{x^2}\right)$

$=16\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{4}}{\frac{x^2}{16}}\right)\times\frac{1}{16}\times\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{8}}{\frac{x^2}{64}}\right)\times\frac{1}{64}$

$=16\times\frac{1}{16}\times\frac{1}{64}\lim_{x\to 0}\left(\frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2\times\lim_{x\to 0}\left(\frac{\sin\frac{x}{8}}{\frac{x}{8}}\right)^2$

$=\frac{1}{64}\times 1\times 1=\frac{1}{64}$

Question 28

Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. Then reflection of R in the xy-plane has co-ordinates

Options:

A. $(2,-4,-7)$

B. $(2,-4,7)$

C. $(-2,4,7)$

D. $(2,4,7)$

Answer: A

Solution:

Let $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}=\lambda$

$\Rightarrow x=3+\lambda, y=3\lambda-1, \mathrm{z}=-\lambda+6$

Let $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}=\mu$

$\Rightarrow x=7\mu-5, y=-6\mu+2, z=4\mu+3$

Both the given lines intersect each other.

So, $\lambda+3=7\mu-5$

$\Rightarrow 7\mu-\lambda=8$ .... (i)

Also, $3\lambda - 1 = -6\mu + 2$

$\Rightarrow 6\mu + 3\lambda = 3$ .... (ii)

From (i) and (ii), we get

$\mu = 1, \lambda = -1$
i.e., $x = 2, y = -4, z = 7$

$\therefore$ Co-ordinates of the intersection of the given lines are $R(2, -4, 7)$

Hence, reflection of R in the xy-plane is $(2, -4, -7)$.

Question 29

The value of $\int \frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}}$ is

Options:

A. $2\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + \mathrm{c}$, where c is a constant of integration.

B. $2\sqrt{2 + \frac{2}{x^2} + \frac{1}{x^4}} + c$, where c is a constant of integration.

C. $\frac{1}{2}\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + \mathrm{c}$, where c is a constant of integration.

D. $2\sqrt{2 - \frac{2}{x^2} - \frac{1}{x^4}} + c$, where c is a constant of integration.

Answer: C

Solution:

Let $\mathrm{I} = \int \frac{(x^2-1)\mathrm{d}x}{x^3\sqrt{2x^4-2x^2+1}}$

$= \int \frac{(x^2-1)\mathrm{d}x}{x^3 \cdot x^2\sqrt{\left(2-\frac{2}{x^2}+\frac{1}{x^4}\right)}}$

$= \int \frac{\left(\frac{x^2-1}{x^5}\right)\mathrm{d}x}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}$

Put $2-\frac{2}{x^2}+\frac{1}{x^4} = \mathrm{t}$

$\Rightarrow \left(\frac{4}{x^3}-\frac{4}{x^5}\right)\mathrm{d}x = \mathrm{dt}$

$\Rightarrow \frac{x^2-1}{x^5}\,\mathrm{d}x = \frac{\mathrm{dt}}{4}$

$\therefore \quad I = \int \frac{\frac{dt}{4}}{\sqrt{t}}$

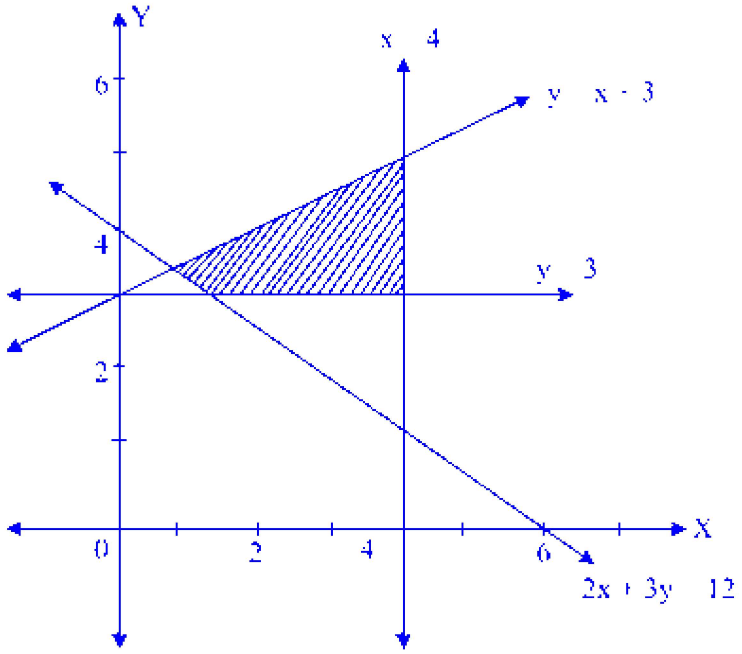
$= \frac{1}{4}\int \mathrm{t}^{\frac{-1}{2}}\mathrm{dt}$

$= \frac{1}{4}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+c$

$\therefore \quad \mathrm{I} = \frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+\mathrm{c}$

Question 30

The shaded area in the figure given below is a solution set of a system of inequations. The minimum value of objective function $3x+5y$, subject to the linear constraints given by this system of inequations is



Options:

A. 19.5

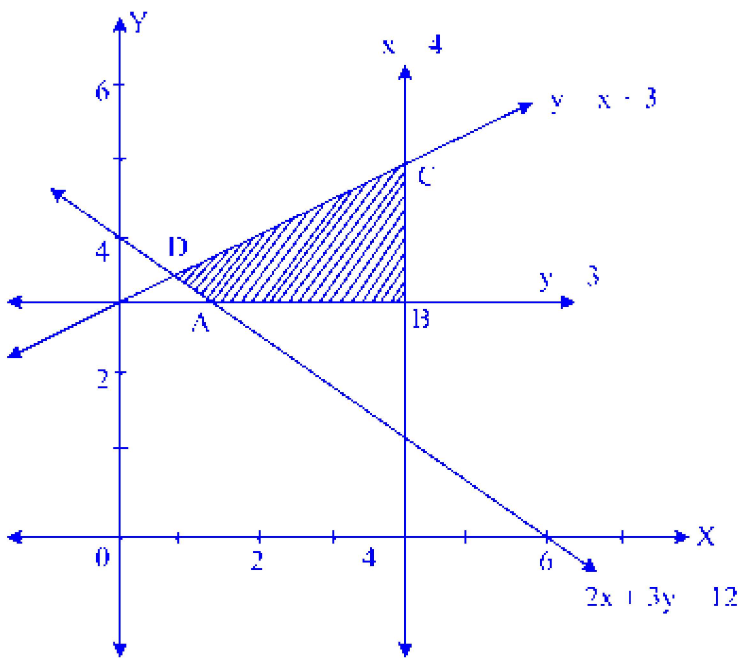
B. 21

C. 15

D. 19.8

Answer: A

Solution:



Let the corner points of the feasible region be A, B, C, D.

Solving equations $y = 3$ and $2x + 3y = 12$, we get

$\mathrm{A} = (1.5, 3)$

Similarly,

$\mathrm{B} = (4, 3)$

$\mathrm{C} = (4, 7)$

$\mathrm{D} = \left(\frac{3}{5}, \frac{18}{5}\right)$

Let $Z = 3x + 5y$

$\therefore$ Value of Z at point $A = 19.5$

Value of Z at point $B = 27$

Value of Z at point $C = 47$

Value of Z at point $D = \frac{99}{5}$

$\therefore$ The minimum value of Z is 19.5.

Question 31

If $\overline{\mathrm{a}} = \mathrm{m}\overline{\mathrm{b}} + \mathrm{nc}$, where $\overline{\mathrm{a}} = 4\hat{\mathrm{i}} + 13\hat{\mathrm{j}} - 18\hat{\mathrm{k}}, \overline{\mathrm{b}} = \hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}, \overline{\mathrm{c}} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 4\hat{\mathrm{k}}$, then $\mathrm{m} + \mathrm{n} =$

Options:

A. 1

B. 2

C. 3

D. -1

Answer: A

Solution:

Given:

$\bar{a} = 4\hat{i} + 13\hat{j} - 18\hat{k}$
$\bar{b} = \hat{i} - 2\hat{j} + 3\hat{k}$
$\bar{c} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

Also, $\overline{\mathrm{a}} = \mathrm{m}\overline{\mathrm{b}} + \mathrm{n}\overline{\mathrm{c}}$

$$\Rightarrow 4\hat{i} + 13\hat{j} - 18\hat{k} = m(\hat{i} - 2\hat{j} + 3\hat{k}) + n(2\hat{i} + 3\hat{j} - 4\hat{k})$$
$$\Rightarrow 4\hat{i} + 13\hat{j} - 18\hat{k}$$
$$= (m + 2n)\hat{i} + (-2m + 3n)\hat{j} + (3m - 4n)\hat{k}$$

Comparing, we get

$m + 2n = 4$ and $-2m + 3n = 13$

Solving above equations, we get $\mathrm{m} = -2\$and\$\mathrm{n} = 3$

$\therefore \quad \mathrm{m} + \mathrm{n} = -2 + 3 = 1$

Question 32

In a game, 3 coins are tossed. A person is paid ₹ 7 /-, if he gets all heads or all tails; and he is supposed to pay ₹ 3 /-, if he gets one head or two heads. The amount he can expect to win on an average per game is ₹

Options:

A. -0.5

B. 0.5

C. 1

D. -1

Answer: A

Solution:

In a game, 3 coins are tossed,

$P($ getting all heads or all tails$) = \frac{2}{8} = \frac{1}{4}$

$P($getting one head or two heads$) = \frac{6}{8} = \frac{3}{4}$

Let X : number of rupees the person gets.

$P(X=7)=\frac{1}{4}$
$P(X=-3)=\frac{3}{4}$

The amount he can expect to win $=$ Mean

$$
\begin{aligned}
&=\sum x_i p_i \\
&=7\left(\frac{1}{4}\right)+-3\left(\frac{3}{4}\right) \\
&=-0.5
\end{aligned}
$$

Question 33

$\int \mathrm{e}^x\left(1-\cot x+\cot ^2 x\right) \mathrm{d} x=$

Options:

A. $\mathrm{e}^x \cdot \cot x+\mathrm{c}$, where c is a constant of integration.

B. $\mathrm{e}^x \cdot \operatorname{cosec} x+\mathrm{c}$, where c is a constant of integration.

C. $-\mathrm{e}^x \cdot \cot x+\mathrm{c}$, where c is a constant of integration.

D. $-\mathrm{e}^x \cdot \operatorname{cosec} x+\mathrm{c}$, where c is a constant of integration.

Answer: C

Solution:

$$
\begin{aligned}
& \int \mathrm{e}^x\left(1-\cot x+\cot ^2 x\right) \mathrm{d} x \\
& =\int \mathrm{e}^x\left(1+\cot ^2 x-\cot x\right) \mathrm{d} x \\
& =\int \mathrm{e}^x\left(-\cot x+\operatorname{cosec}^2 x\right) \mathrm{d} x \\
& =\mathrm{e}^x(-\cot x)+\mathrm{c} \\
& \quad \ldots\left[\because \int \mathrm{e}^x\left[\mathrm{f}(x)+\mathrm{f}^{\prime}(x)\right] \mathrm{d} x=\mathrm{e}^x \mathrm{f}(x)+\mathrm{c}\right] \\
& =-\mathrm{e}^x \cdot \cot x+\mathrm{c}
\end{aligned}
$$

Question 34

The parametric equations of the circle $x^2+y^2+2x-4y-4=0$ are

Options:

A. $x=-1+3\cos\theta, y=2+3\sin\theta$

B. $x=1+3\cos\theta, y=-2+3\sin\theta$

C. $x=-1+3\sin\theta, y=-2+3\cos\theta$

D. $x=1+3\sin\theta, y=-2+3\cos\theta$

Answer: A

Solution:

Given equation of circle is

$$x^2+y^2+2x-4y-4=0$$
$$\Rightarrow (x^2+2x+1)+(y^2-4y+4)-4-5=0$$
$$\Rightarrow (x+1)^2+(y-2)^2=3^2$$

$\therefore$ Parametric equation of circle is

$x=-1+3\cos\theta, y=2+3\sin\theta$

Question 35

In a triangle ABC, with usual notations, if $c=4$, then value of $(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}$ is

Options:

A. 4

B. 16

C. 9

D. 25

Answer: B

Solution:

$$(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2}$$
$$= \left(a^2 - 2ab + b^2\right) \cos^2 \frac{C}{2} + \left(a^2 + 2ab + b^2\right) \sin^2 \frac{C}{2}$$
$$= \left(a^2 + b^2\right) \left(\cos^2 \frac{C}{2} + \sin^2 \frac{C}{2}\right)$$
$$-2ab \cos^2 \frac{C}{2} + 2ab \sin^2 \frac{C}{2}$$
$$= a^2 + b^2 - 2ab \left(\cos^2 \frac{C}{2} - \sin^2 \frac{C}{2}\right)$$
$$= a^2 + b^2 - 2ab \cdot \cos^2$$
$$= a^2 + b^2 - \left(a^2 + b^2 - c^2\right) \quad \ldots [By \text{ Cosine rule }]$$
$$= c^2 = 4^2 = 16$$

Question 36

$$\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} =$$

Options:

A. $\frac{1}{3\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{2}{3\sqrt{3}}$

D. $\frac{-2}{3\sqrt{3}}$

Answer: C

Solution:

$$\lim_{x \to a} \frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$$
$$= \lim_{x \to a} \frac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{a+2x}+\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{3a+x}-2\sqrt{x})(\sqrt{3a+x}+2\sqrt{x})(\sqrt{a+2x}+\sqrt{3x})}$$
$$= \lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$
$$= \frac{(\sqrt{3a+a}+2\sqrt{a})}{3(\sqrt{a+2a}+\sqrt{3a})}$$
$$= \frac{1}{3} \cdot \frac{(2\sqrt{a}+2\sqrt{a})}{(\sqrt{3a}+\sqrt{3a})}$$
$$= \frac{4\sqrt{a}}{6\sqrt{3a}}$$
$$= \frac{2}{3\sqrt{3}}$$

Question 37

If $B = \begin{bmatrix} 3 & \alpha & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

Options:

A. 1

B. 0

C. -1

D. -2

Answer: A

Solution:

$$\text{Using } |\text{adj } A| = |A|^{n-1}$$
$$But \quad B = \text{Adj}(A) \quad \ldots . [\text{Given}]$$
$$\therefore \quad |B| = |A|^2$$
$$\Rightarrow \quad \begin{vmatrix} 3 & \alpha & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{vmatrix} = |A|^2$$
$$\Rightarrow \quad 24 - 4\alpha - 4 = 4^2$$
$$\Rightarrow \quad 20 - 4\alpha = 16$$
$$\Rightarrow \quad \alpha = 1$$

Question 38

If $x = 3\tan \mathrm{t}$ and $y = 3\sec \mathrm{t}$, then the value of $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ at $\mathrm{t} = \frac{\pi}{4}$ is

Options:

A. $\frac{-1}{6\sqrt{2}}$

B. $\frac{1}{6\sqrt{2}}$

C. $\frac{1}{3\sqrt{2}}$

D. $\frac{3}{2\sqrt{2}}$

Answer: B

Solution:

$x = 3\tan t$

$\therefore \quad \frac{dx}{dt} = 3\sec^2 t$

$y = 3\sec t$

$\therefore \quad \frac{dy}{dt} = 3\sec t\tan t$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\sec t\tan t}{3\sec^2 t} = \sin t$

$\therefore \quad \frac{d^2y}{dx} = \frac{d}{dt}(\sin t)\cdot\frac{dt}{dx}$

$= \cos t \times \frac{1}{3\sec^2 t}$

$\therefore \quad \frac{d^2y}{dx^2} = \frac{\cos^3 t}{3}$

$\therefore \quad \left(\frac{d^2y}{dx^2}\right)_{(1-\frac{\pi}{4})} = \frac{\left(\cos\frac{\pi}{4}\right)^3}{3} = \frac{1}{6\sqrt{2}}$

Question 39

A fair die is tossed twice in succession. If X denotes the number of sixes in two tosses, then the probability distribution of X is given by

Options:

A.

X $=x$	0	1	2
P(X $=x$)	$\frac{25}{36}$	$\frac{1}{36}$	$\frac{5}{18}$

B.

X $=x$	0	1	2
P(X $=x$)	$\frac{5}{18}$	$\frac{1}{36}$	$\frac{25}{36}$

C.

X $=x$	0	1	2
P(X $=x$)	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

D.

X $=x$	0	1	2
P(X $=x$)	$\frac{5}{18}$	$\frac{25}{36}$	$\frac{1}{36}$

Answer: C

Solution:

X can take values 0,1 and 2 .

$\mathrm{P}(\mathrm{X}=0)=$ Probability of not getting six $=\frac{25}{36}$

$P(X=1)=$ Probability of getting one six

$=\frac{10}{36}=\frac{5}{18}$

$P(X=2)=$ Probability of getting two sixes $=\frac{1}{36}$

The probability distribution of X is

X $=x$	0	1	2
P(X $=x$)	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

Question 40

The negation of the statement pattern $\sim s \vee(\sim r \wedge s)$ is equivalent to

Options:

A. $s \wedge r$

B. $s \wedge(r \wedge \sim s)$

C. $s \wedge \sim r$

D. $s \vee(r \vee \sim s)$

Answer: A

Solution:

$\sim(\sim s \vee(\sim r \wedge s))$
$\equiv s \wedge \sim(\sim r \wedge s)$ ... [De Morgan's law]
$\equiv s \wedge(r \vee \sim s)$ ... [De Morgan's law]
$\equiv(s \wedge r) \vee(s \wedge \sim s)$ ... [Distributive law]
$\equiv(s \wedge r) \vee F$ ... [Complement law]
$\equiv s \wedge r$ ... [Identity law]

Question 41

If the volume of tetrahedron, whose vertices are with position vectors $\hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+7\hat{k}, 5\hat{i}-\hat{j}+\lambda\hat{k}$ and $7\hat{i}-4\hat{j}+7\hat{k}$ is 11 cubic units, then value of λ is

Options:

A. 4

B. 5

C. 7

D. 6

Answer: C

Solution:

Let $\bar{a}=\hat{i}-6\hat{j}+10\hat{k}$,
$\overline{\mathrm{b}}=-\hat{\mathrm{i}}-3\hat{\mathrm{j}}+7\hat{\mathrm{k}}$,
$\overline{\mathrm{c}}=5\hat{\mathrm{i}}-\hat{\mathrm{j}}+\lambda\hat{\mathrm{k}}$,
$\overline{\mathrm{d}}=7\hat{\mathrm{i}}-4\hat{\mathrm{j}}+7\hat{\mathrm{k}}$
$\therefore \quad \overline{\mathrm{AB}}=-2\hat{\mathrm{i}}+3\hat{\mathrm{j}}-3\hat{\mathrm{k}}$
$\overline{\mathrm{AC}}=4\hat{\mathrm{i}}+5\hat{\mathrm{j}}+(\lambda-10)\hat{\mathrm{k}}$
$\overline{\mathrm{AD}}=6\hat{\mathrm{i}}+2\hat{\mathrm{j}}-3\hat{\mathrm{k}}$
$\therefore \quad$ Volume of tetrahedron $=\frac{1}{6}[\overline{\mathrm{AB}} \quad \overline{\mathrm{AC}} \quad \overline{\mathrm{AD}}]$

$$\Rightarrow 11=\frac{1}{6}\left|\begin{array}{ccc}-2 & 3 & -3 \\ 4 & 5 & \lambda-10 \\ 6 & 2 & -3\end{array}\right|$$

$$\Rightarrow 11=\frac{1}{6}\{-2(-15-2\lambda+20)-3(-12-6\lambda+60)-3(8-30)\}$$

$\Rightarrow \lambda=7$

Question 42

The argument of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}, i=\sqrt{-1}$ is

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: C

Solution:

$$\text{Let } z=\frac{1+i\sqrt{3}}{\sqrt{3}+i}$$

$$=\frac{(1+i\sqrt{3})(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)}$$

$$\therefore \quad z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$$

Argument of

$$z=\tan^{-1}\left(\frac{b}{a}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$$

$$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$=\frac{\pi}{6}$$

Question 43

If $y=\tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right)+\tan^{-1}\left(\frac{4+2\log x}{1-8\log x}\right)$, then $\frac{dy}{dx}$ is

Options:

A. 0

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. 1

Answer: A

Solution:

$$y=\tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log\left(ex^2\right)}\right)+\tan^{-1}\left(\frac{4+2\log x}{1-8\log x}\right)$$
$$=\tan^{-1}\left(\frac{\log e-\log x^2}{\log e+\log x^2}\right)+\tan^{-1}(4)+\tan^{-1}(2\log x)$$
$$=\tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right)+\tan^{-1}(4)+\tan^{-1}(2\log x)$$
$$=\tan^{-1}(1)-\tan^{-1}(2\log x)+\tan^{-1}(4)+\tan^{-1}(2\log x)$$
$$\therefore\quad y=\tan^{-1}(1)+\tan^{-1}(4)$$
$$\therefore\quad \frac{dy}{dx}=0$$

Question 44

If the surface area of a spherical balloon of radius 6 cm is increasing at the rate $2\text{ cm}^2/\text{sec}$, then the rate of increase in its volume in cm^3/sec is

Options:

A. 16

B. 6

C. 12

D. 8

Answer: B

Solution:

Surface area, $S = 4\pi r^2$

$$\therefore \quad \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\Rightarrow \quad 2 = 8\pi r \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{1}{4\pi r} \quad \text{... (i)}$$

Volume, $V = \frac{4}{3}\pi r^3$

$$\therefore \quad \frac{dV}{dt} = \frac{4}{3} \times 3\pi r^2 \times \frac{dr}{dt}$$

$$= 4\pi r^2 \times \frac{1}{4\pi r} \quad \ldots \text{[From (i)]}$$

$$= r$$

$$= 6 \text{ cm}^3/\text{sec}$$

Question 45

The value of α, so that the volume of parallelopiped formed by $\hat{i} + \alpha\hat{j} + \hat{k}, \hat{j} + \alpha\hat{k}$ and $\alpha\hat{i} + \hat{k}$ becomes minimum, is

Options:

A. -3

B. 3

C. $\frac{1}{\sqrt{3}}$

D. $-\frac{1}{\sqrt{3}}$

Answer: C

Solution:

$$\text{Volume of parallelopiped} = \begin{vmatrix} 1 & \alpha & 1 \\ 0 & 1 & \alpha \\ \alpha & 0 & 1 \end{vmatrix}$$

$$\therefore \quad V = 1 + \alpha^3 - \alpha$$

For maxima or minima,

$$\frac{dV}{d\alpha} = 0$$
$$\Rightarrow 3\alpha^2 - 1 = 0$$
$$\Rightarrow \alpha = \pm\frac{1}{\sqrt{3}}$$

Now, $\frac{d^2V}{d\alpha^2} = 6\alpha$

For $\alpha = \frac{1}{\sqrt{3}}$,

$\frac{d^2\ V}{dx^2} > 0$

$\therefore \quad V$ is minimum at $\alpha = \frac{1}{\sqrt{3}}$.

Question 46

In a certain culture of bacteria, the rate of increase is proportional to the number of bacteria present at that instant. It is found that there are 10,000 bacteria at the end of 3 hours and 40,000 bacteria at the end of 5 hours, then the number of bacteria present in the beginning are

Options:

A. 1250

B. 1200

C. 1350

D. 1300

Answer: A

Solution:

Let x be the number of bacteria present at time t.

$\therefore \quad \frac{dx}{dt} \propto x$
$\therefore \quad \frac{dx}{dt} = kx$
$\therefore \quad \frac{dx}{x} = kdt$

Integrating on both sides, we get

$\log x = kt + c$ .... (i)

When $t = 3, x = 10,000$

Equation (i) becomes

$\log(10,000) = 3\text{k} + \text{c}$ .... (ii)

When $\text{t} = 5, x = 40,000$

Equation (i) becomes

$\log(40,000) = 5k + c$ .... (iii)

Subtracting (ii) from (iii), we get

$\text{k} = \log 2$

From equation (ii),

$\log(10,000) = 3\log 2 + c$

$\therefore \quad c = \log(1250)$

Now, Initially $\text{t} = 0$

From (i),

$$\begin{aligned} & \log x = \text{k} \times 0 + \log(1250) \\ \therefore \quad & \log x = \log 1250 \\ \therefore \quad & x = 1250 \end{aligned}$$

Question 47

If x, y, z are in A.P. and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in A.P., then

Options:

A. $x = y = z$

B. $2x = 3y = 6z$

C. $6x = 3y = 2z$

D. $6x = 4y = 3z$

Answer: A

Solution:

Given, x, y, z are in A.P.

$\therefore \quad 2y = x + z$ ..... (i)

Also,

$\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in A.P.

$$\therefore \quad 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \quad \frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$

$$\Rightarrow \quad \frac{2y}{1-y^2} = \frac{2y}{1-xz} \quad \ldots \text{[From (i)]}$$

$$\Rightarrow \quad 1-y^2 = 1-xz$$

$$\Rightarrow \quad y^2 = xz$$

$\therefore$ x, y, z are in G.P. ..... (ii)

From (i) and (ii), we get

$x = y = z$

Question 48

Mean and variance of six observations are 8 and 16 respectively. If each observation is multiplied by 3, then new variance of the resulting observations is

Options:

A. 16

B. 48

C. 24

D. 144

Answer: D

Solution:

When each item of a data is multiplied by λ, variance is multiplied by λ^2.

$$\begin{aligned} \therefore \quad \text{New variance} &= 3^2 \times 16 \\ &= 9 \times 16 \\ &= 144 \end{aligned}$$

Question 49

The differential equation of all circles, passing through the origin and having their centres on the X-axis, is

Options:

A. $y^2 = x^2 + xy\frac{dy}{dx}$

B. $x^2 = y^2 + 2xy\frac{dy}{dx}$

C. $y^2 = x^2 + 2xy\frac{dy}{dx}$

D. $x^2 = y^2 - xy\frac{dy}{dx}$

Answer: C

Solution:

The system of circles which passes through origin and whose centre lies on X-axis is

$x^2 + y^2 - 2bx = 0$ .... (i)

Differentiating w.r.t x, we get

$2x + 2y\frac{dy}{dx} = 2$ b ..... (ii)

Substituting (ii) in (i), we get

$$x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$
$$\Rightarrow y^2 - x^2 - 2xy\frac{dy}{dx} = 0$$
$$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$$

Question 50

$$\int_0^1 \cos^{-1} x dx =$$

Options:

A. -1

B. 0

C. 1

D. 2

Answer: C

Solution:

$$
\begin{aligned}
\text{Let} \quad \mathrm{I} &= \int_0^1 \left(\cos^{-1} x\right)(1) \mathrm{d}x \\
&= \left[\cos^{-1} x \cdot x - \int \left(\frac{-1}{\sqrt{1-x^2}} \cdot x\right) \mathrm{d}x\right]_0^1 \\
&= \left[x \cdot \cos^{-1} x + \int \frac{x}{\sqrt{1-x^2}} \mathrm{~d}x\right]_0^1 \\
&= \left[x \cos^{-1} x - \sqrt{1-x^2}\right]_0^1 \\
&= \left[1 \cdot \cos^{-1}(1) - \sqrt{1-(1)^2}\right] - \left[0 \cdot \cos^{-1}(0) - \sqrt{1-0^2}\right] \\
&= 0 - (-1) \\
&= 1
\end{aligned}
$$

Chemistry

Question 51

What is the calculated value of spin only magnetic moment in terms of BM if only one unpaired electron is present in a species?

Options:

A. 2.76

B. 2.84

C. 2.2

D. 1.73

Answer: D

Solution:

No. of unpaired $e^{-} = 1$

$\mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = \sqrt{3} = 1.73$ BM

Question 52

Which of the following is tertiary allylic alcohol?

Options:

A. Prop-2-en-1-ol

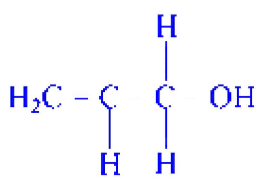
B. But-3-en-2-ol

C. 2-Methylprop-2-en-ol

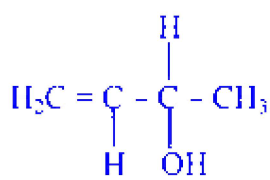
D. 2-Methylbut-3-en-2-ol

Answer: D

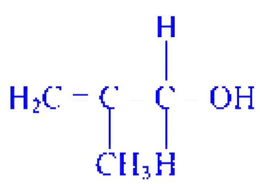
Solution:



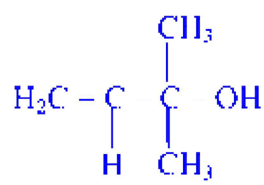
Prop-2-en-1-ol



But-3-en-2-ol



2-Methylprop-2-en-ol



2-Methylbut-3-en-2-ol

2-Methylbut-3-en-2-ol is a tertiary allylic alcohol.

Question 53

Identify the solvent used in bromination of phenol to obtain 2,4,6-tribromophenol.

Options:

A. Chloroform

B. Water

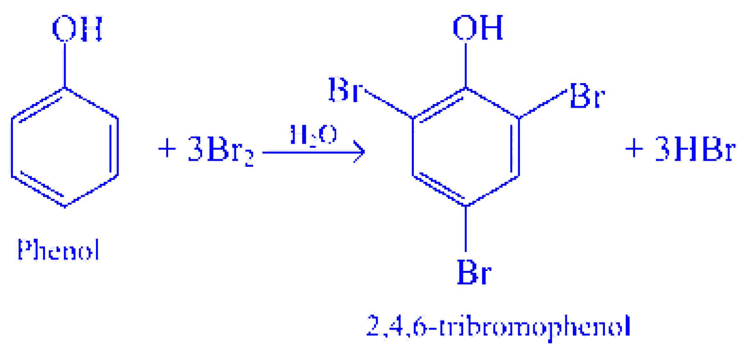
C. Carbon tetrachloride

D. Carbon disulphide

Answer: B

Solution:

Phenol reacts with aqueous solution of bromine to give 2,4,6-tribromophenol.



If the bromination of phenol is carried out in a solvent of lower polarity than water, such as CS_2, $CHCl_3$ or CCl_4, a mixture of ortho- and para-bromophenol is formed.

Question 54

How many moles of nitrogen atoms are present in 8 g of ammonium nitrate?

(Molar mass of ammonium nitrate = 80)

Options:

A. 0.1 mol

B. 0.2 mol

C. 0.4 mol

D. 0.8 mol

Answer: B

Solution:

To find out the number of moles of nitrogen atoms present in 8 g of ammonium nitrate NH_4NO_3, we first need to calculate the number of moles of ammonium nitrate itself. We can do this using its molar mass.

The molar mass of ammonium nitrate is given as 80 g/mol.

Using the formula

Number of moles $= \frac{\text{Mass}}{\text{Molar Mass}}$

We substitute the mass of ammonium nitrate and its molar mass and get

Number of moles of ammonium nitrate $= \frac{8\text{ g}}{80\text{ g/mol}}$ Number of moles of ammonium nitrate $= 0.1$ mol

Now, in one molecule of ammonium nitrate, there are two nitrogen atoms (one in the ammonium ion NH_4^+ and one in the nitrate ion NO_3^-.

To find the total number of moles of nitrogen atoms, we then need to multiply the number of moles of ammonium nitrate by the number of nitrogen atoms per molecule of ammonium nitrate.

Number of moles of nitrogen atoms $=$ Number of moles of ammonium nitrate $\times$ Number of nitrogen atoms per molecule
Number of moles of nitrogen atoms $= 0.1$ mol $\times\ 2$ Number of moles of nitrogen atoms $= 0.2$ mol

Therefore, 8 g of ammonium nitrate contains 0.2 mol of nitrogen atoms. The correct answer is:

Option B 0.2 mol

Question 55

Identify the elements undergoing reduction and oxidation respectively in the following redox reaction.

$$3H_3AsO_{3(aq)} + BrO^-_{3(aq)} \rightarrow Br^-_{(aq)} + 3H_3AsO_4$$

Options:

A. As and O

B. Br and As

C. As and Br

D. Br and O

Answer: B

Solution:

The oxidation number of Br decreases from +5 to -1 and that of As increases from +3 to +5.

Hence, Br undergoes reduction and As undergoes oxidation.

Question 56

Which of the following is tricarboxylic acid?

Options:

A. Propionic acid

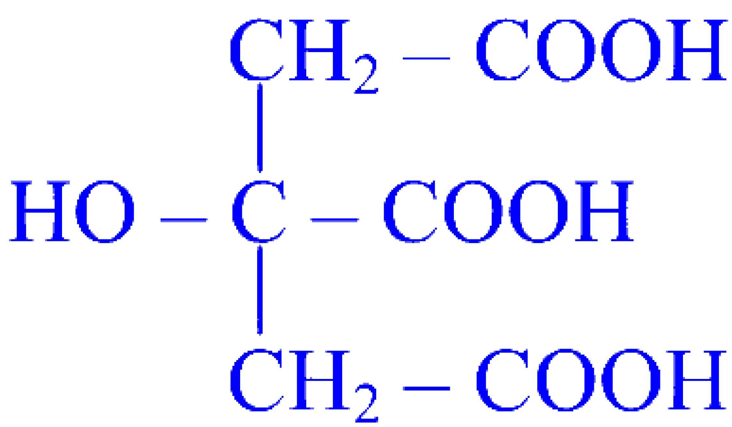
B. Oxalic acid

C. Malonia acid

D. Citric acid

Answer: D

Solution:



Citric acid
(Tricarboxylic acid)

Question 57

Find the radius of an atom in fcc unit cell having edge length 393pm.

Options:

A. 196.51 pm

B. 170.22 pm

C. 78.63 pm

D. 138.93 pm

Answer: D

Solution:

For fcc crystal structure,

$$4r = \sqrt{2}\text{a}$$
$$\therefore \quad r = \frac{\sqrt{2}\text{a}}{4} = \frac{1.414 \times 393}{4} = 138.93 \text{ pm}$$

Question 58

Which from following is a CORRECT decreasing order of ionization enthalpies of different elements?

Options:

A. $\text{S} > \text{Se} > \text{Te} > \text{Po}$

B.

$\text{Te} > \text{Po} > \text{S} > \text{Se}$

C. $\text{S} > \text{Te} > \text{Po} > \text{Se}$

D. $\text{Te} > \text{Po} > \text{Se} > \text{S}$

Answer: A

Solution:

The ionization enthalpy decreases down the group due to increase in the atomic size. The correct decreasing order of ionization enthalpies is:

$S > Se > Te > Po$

Question 59

A gas absorbs 200 J heat and expands by $500\ cm^3$ against a constant external pressure $2 \times 10^5\ N\ m^{-2}$. What is the change in internal energy?

Options:

A. 800 J

B. -750 J

C. 100 J

D. -150 J

Answer: C

Solution:

$$\begin{aligned} \Delta V &= 500\ cm^3 = 0.5\ dm^3 \\ P_{ext} &= 2 \times 10^5\ N\ m^{-2} = 2 bar \quad \left(\text{since, } 1 \times 10^5\ N\ m^{-2} = 1 bar\right) \\ W &= -P_{ext} \times \Delta V \\ &= -2 \times (0.5) \\ &= -1 dm^3\ bar \\ \therefore \quad W &= -100\ J \quad \left(\text{since, } 1 dm^3 bar = 100\ J\right) \end{aligned}$$

According to first law of thermodynamics,

$\Delta U = Q + W = 200\ J - 100\ J = +100\ J$

Question 60

Identify a copolymer from following.

Options:

A. Natural rubber

B. Polypropene

C. PVC

D. Terylene

Answer: D

Solution:

Terylene is a copolymer while natural rubber, polypropene and PVC are homopolymers.

Question 61

Find the average rate of formation of $NO_{2(g)}$, in following reaction.

$$2 N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$$
$$\left[-\frac{\Delta [N_2O_5]}{\Delta t}\right] = x \text{ mol dm}^{-3} \text{ s}^{-1}$$

Options:

A. x mol dm^{-3} s^{-1}

B. $\frac{x}{2}$ mol dm^{-3} s^{-1}

C. $2x$ mol dm^{-3} s^{-1}

D. $4x$ mol dm^{-3} s^{-1}

Answer: C

Solution:

$$\frac{-\Delta [N_2O_5]}{\Delta t} = x \text{ moldm}^{-3} \text{ s}^{-1}$$
$$2 N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$$

Average rate of reaction

$$= -\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t} = \frac{1}{4}\frac{\Delta[NO_2]}{\Delta t} = \frac{\Delta[O_2]}{\Delta t}$$

$$\therefore \frac{\Delta[NO_2]}{\Delta t} = -\frac{4}{2}\frac{\Delta[N_2O_5]}{\Delta t} = \frac{4}{2} \times x$$

$$= 2x \text{ moldm}^{-3}\text{ s}^{-1}$$

$\therefore$ Average rate of formation of $NO_{2(g)} = 2x$ mol dm^{-3} s^{-1}

[Note: In the question, $\frac{\Delta[N_2O_5]}{\Delta t}$ is changed to $\frac{-\Delta[N_2O_5]}{\Delta t}$ to apply appropriate textual concepts.]

Question 62

Calculate the rate constant for the first order reaction, $A \rightarrow B$ if the rate of reaction is 5.4×10^{-6} mol dm^{-3} s^{-1} and $[A] = 0.3M$.

Options:

A. 1.8×10^{-5} s^{-1}

B. 1.5×10^{-5} s^{-1}

C. 2.1×10^{-5} s^{-1}

D. 2.4×10^{-5} s^{-1}

Answer: A

Solution:

For the first order reaction, $A \longrightarrow B$,

$$\text{Rate} = k[A]$$

$$\therefore k = \frac{\text{Rate}}{[A]} = \frac{5.4 \times 10^{-6}\text{ mol dm}^{-3}\text{ s}^{-1}}{0.3\text{ mol dm}^{-3}}$$

$$= 1.8 \times 10^{-5}\text{ s}^{-1}$$

Question 63

Identify the product obtained when benzonitrile is reduced by stannous chloride in presence of hydrochloric acid followed by acid hydrolysis.

Options:

A. Benzal chloride

B. Benzoyl chloride

C. Benzophenone

D. Benzaldehyde

Answer: D

Solution:

This is Stephen reaction.

$$\underset{\text{Benzonitrile}}{C_6H_5 - C \equiv N} + 2[H] \xrightarrow{SnCl_2\ .\ HCl} \underset{\text{Benzanimine hydrochloride}}{C_6H_5 - HC = NH.HCl}$$

$$\xrightarrow{H_3O^+} \underset{\text{Benzaldehyde}}{C_6H_5 - CHO} + NH_4Cl$$

Question 64

Find solubility in terms of $\mathrm{mol\ L^{-1}}$ if solubility product of silver bromide is 6.4×10^{-13}.

Options:

A. $4.0 \times 10^{-5}\ \mathrm{mol\ L^{-1}}$

B. $8.0 \times 10^{-7}\ \mathrm{mol\ L^{-1}}$

C. $7.5 \times 10^{-5}\ \mathrm{mol\ L^{-1}}$

D. $6.4 \times 10^{-4}\ \mathrm{mol\ L^{-1}}$

Answer: B

Solution:

$AgBr_{(s)} \rightleftharpoons Ag^{+}_{(aq)} + Br^{-}_{(aq)}$

$\therefore \quad x = 1, y = 1$

$K_{sp} = x^x y^y S^{x+y} = (1)^1 (1)^1 S^{1+1} = S^2$

$\therefore \quad S = \sqrt{K_{sp}} = \sqrt{6.4 \times 10^{-13}} = \sqrt{64 \times 10^{-14}}$

$= 8 \times 10^{-7}$ mol L^{-1}

Question 65

What happens when solution of an electrolyte is diluted?

Options:

A. Both $\wedge$ and k increases

B. Both $\wedge$ and k decreases

C. $\wedge$ increases and k decreases

D. $\wedge$ decreases and k increases

Answer: C

Solution:

The molar conductivity ($\wedge$) of an electrolyte increases on dilution while conductivity (k) decreases on dilution.

Question 66

The solubility of a gas in a liquid is directly proportional to the pressure of the gas over the solution. Identify the law for this statement.

Options:

A. Henry's law

B. Raoult's law

C. Dalton's law

D. Avogadro's law

Answer: A

Question 67

Weak acid HX has dissociation constant 1×10^{-5}. Calculate the percent dissociation in its $0.1 M$ solution.

Options:

A. 2.2%

B. 3.5%

C. 4.2%

D. 1.0%

Answer: D

Solution:

$$K_a = 1 \times 10^{-5}, c = 0.1M$$
$$K_a = \alpha^2 c$$
$$\therefore \quad \alpha = \sqrt{\frac{K_a}{c}} = \sqrt{\frac{1 \times 10^{-5}}{0.1}} = \sqrt{1 \times 10^{-4}} = 0.01$$

$$\alpha = \frac{\text{Percent dissociation}}{100}$$
$$\text{Percent dissociation } = \alpha \times 100 = 0.01 \times 100 = 1.0\%$$

Question 68

Calculate the pH of a buffer solution containing 0.01 M salt and 0.004 M weak acid.

$(pK_a = 4.762)$

Options:

A. 4.36

B. 4.76

C. 5.16

D. 5.36

Answer: C

Solution:

$$\begin{aligned}pH &= pK_a + \log_{10}\frac{[\text{ Salt }]}{[\text{ Acid }]}\\ &= 4.762 + \log_{10}\frac{0.01}{0.004}\\ &= 4.762 + (\log_{10} 5 - \log_{10} 2)\\ &= 4.762 + (0.699 - 0.3010)\\ &= 4.762 + 0.398\\ &= 5.16\end{aligned}$$

Question 69

Which among the following is vinylic halide?

Options:

A. $CH_2 = CH - CH_2 - X$

B. $CH_3 - CH = CH - X$

C. $CH_3 - C \equiv C - X$

D. $C_6H_5 - CH_2 - X$

Answer: B

Solution:

In vinylic halides, halogen atom is bonded to a sp^2 hybridized carbon atom of aliphatic chain. $CH_3 - CH = CH - X$ is a vinylic halide.

Question 70

Which from following compounds is obtained when carbon dioxide gas bubbled through slaked lime solution?

Options:

A. $CaCO_{3\,(s)}$

B. $CaCl_{2\,(aq)}$

C. $CaSO_{4(s)}$

D. $NaCl_{(aq)}$

Answer: A

Solution:

When carbon dioxide is bubbled through solution of calcium hydroxide (slaked lime), water insoluble solid calcium carbonate is formed.

$Ca(OH)_{2(aq)} + CO_{2(g)} \rightarrow CaCO_{3(s)} + H_2O_{(l)}$

Question 71

Calculate the number of atoms present in unit cell of an element having molar mass $190\ g\ mol^{-1}$ and density $20\ g\ cm^{-3}$.

$$\left[a^3 \cdot N_A = 38\ cm^3\ mol^{-1}\right]$$

Options:

A. 1

B. 2

C. 6

D. 4

Answer: D

Solution:

$$\text{Density } (\rho) = \frac{Mn}{a^3 N_A}$$

$$20 = \frac{190 \times n}{38}$$

$$n = \frac{20 \times 38}{190} = 4$$

Question 72

What is the solubility of a gas in water at $25^{\circ}C$ if partial pressure is 0.18 atm ?

$$\left(K_H = 0.16 \text{ moldm}^{-3} \text{ atm}^{-1}\right)$$

Options:

A. 0.029 mol dm^{-3}

B. 0.022 mol dm^{-3}

C. 0.032 mol dm^{-3}

D. 0.038 mol dm^{-3}

Answer: A

Solution:

$$\begin{aligned} S &= K_H \times P \\ &= 0.16 \times 0.18 \\ &= 0.0288 \text{ mol dm}^{-3} \end{aligned}$$

Question 73

What is the radius of fourth orbit of Be^{+++} ?

Options:

A. 211.6 pm

B. 158.7 pm

C. 52.9 pm

D. 13.2 pm

Answer: A

Solution:

Be^{+++} is a hydrogen-like species having $Z = 4$. Radius of the fourth orbit of Be^{+++}

$= r_4 = \frac{52.9 \times (4)^2}{4} pm = 211.6$ pm

Question 74

What is the number of unpaired electrons in NO molecule?

Options:

A. 0

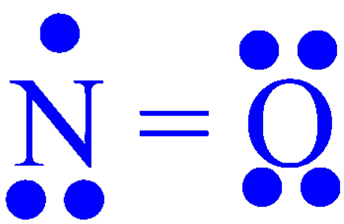
B. 1

C. 2

D. 3

Answer: B

Solution:



No. of unpaired electrons = 1

Question 75

Identify the glycosidic linkage present in lactose.

Options:

A. $\beta-1,4$

B. $\alpha-1,6$

C. $\beta-1,6$

D. $\alpha-1,4$

Answer: A

Solution:

In lactose, the glycosidic linkage is formed between $C-1$ of β-D-galactose and $C-4$ of β-D-glucose.

Question 76

Which from following statements is TRUE about $CH_3CH(NH_2)CH_2COOH$ molecule?

Options:

A. It is an example of amino acids.

B. Its IUPAC name is 3-carboxy-2-amine.

C. The amino group is located at $C-2$.

D. $-NH_2$ group is considered as principal functional group for IUPAC naming.

Answer: A

Solution:

The given molecule is not an amino acid. Its IUPAC name is 3-aminobutanoic acid. The amino substituent is located at C-3 on four carbon chain. $-COOH$ group is considered as principal functional group for IUPAC naming. Hence, statements (A), (B), (C) and (D) are incorrect about the given molecule.

[Note: For the given question, none of the provided options is the correct answer.]

Question 77

Which of the following is obtained as product when ethylene reacts with oxygen in presence of Pd/Al_2O_3 ?

Options:

A. Acetaldehyde

B. Acetic acid

C. Methane

D. Methyl alcohol

Answer: A

Solution:

$$\underset{\text{Ethylene}}{C_2H_{4(g)}} + O_{2(g)} \xrightarrow{Pd/Al_2O_3} \underset{\text{Acetaldehyde}}{2CH_3CHO_{(g)}}$$

Question 78

What is the position of transition elements from Sc to Zn in long form of periodic table?

Options:

A. Group 4 to 13, period-3

B. Group 3 to 12, period-4

C. Group 5 to 14, period-4

D. Group 3 to 12, period-5

Answer: B

Solution:

The transition elements from Scandium (Sc) to Zinc (Zn) in the periodic table are found in the d-block. These elements specifically fall within the groups 3 to 12. In terms of periods, Scandium (Sc), which is the first element in this series, starts at atomic number 21, and this places it in the 4th period of the periodic table. Zinc (Zn), with atomic number 30, is also in the same period.

Therefore, the position of the transition elements from Scandium (Sc) to Zinc (Zn) in the long form of the periodic table is :

Option B : Group 3 to 12, period-4

Question 79

Which from following elements is most abundant on earth?

Options:

A. N

B. C

C. O

D. H

Answer: C

Solution:

The most abundant element on Earth is oxygen (O). It is a major component of the Earth's crust and also a significant part of the atmosphere in the form of molecular oxygen (O_2). Therefore, the correct option is :

Option C : O

Question 80

What is the volume of 1 mole real gas at STP $\left(V_0 = 22.4\ \mathrm{dm}^3\right)$, if compressibility factor of real gas is 1.1 at STP?

Options:

A. 22.40 dm^3

B. 23.64 dm^3

C. 24.64 dm^3

D. 23.50 dm^3

Answer: C

Solution:

$$Z=\frac{V_{\text{real}}}{V_{\text{ideal}}}$$
$$V_{\text{real}} \quad = Z \times V_{\text{ideal}} \quad = 1.1 \times 22.4 = 24.64\ \text{dm}^3$$

Alternate method:

$$\mathrm{Z}=\frac{\mathrm{PV}}{\mathrm{nRT}}$$
$$1.1=\frac{1 \times \mathrm{V}}{1 \times 0.08206 \times 273}$$
$$1.1=\frac{\mathrm{V}}{0.08206 \times 273}$$
$$\therefore \quad \mathrm{V}=24.64\ \mathrm{dm}^3$$

Question 81

Identify the product obtained when isopropyl bromide is reacted with metallic sodium in dry ether.

Options:

A. Isobutane

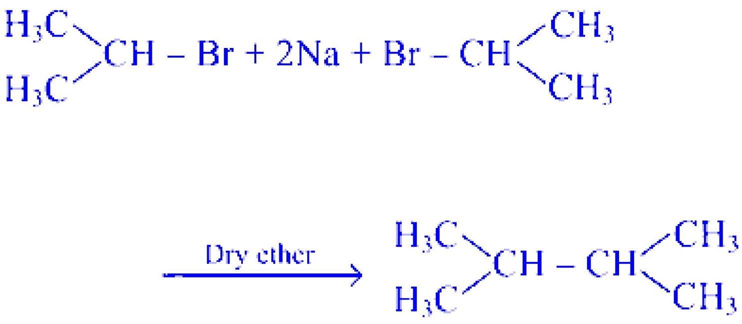
B. Isohexane

C. 2,3-Dimethylbutane

D. 2,3-Dimethylhexane

Answer: C

Solution:



2,3-Dimethylbutane

Question 82

Which of the following is general representation of Grignard reagent?

Options:

A. $R - O - Na$

B. $R - COO - Ag$

C. $R - Mg - X$

D. $R - COOK$

Answer: C

Solution:

Grignard reagents are a class of organomagnesium compounds typically represented by the general formula $R - Mg - X$, where R is an organic group (like an alkyl or aryl group) and X is a halogen (such as chlorine, bromine, or iodine). Therefore, the correct option for the general representation of a Grignard reagent is :

Option C : $R - Mg - X$

Question 83

Which among the following statements is NOT true about high density polythene?

Options:

A. It is obtained from ethene.

B. It needs high pressure (1000-2000 atm) for synthesis.

C. Polymerization occurs in presence of Zieglar-Natta catalyst.

D. Melting point is higher than LDP.

Answer: B

Solution:

HDP is obtained by polymerization of ethene in presence of Zieglar-Natta catalyst at a temperature of 333 K to 343 K and a pressure of $6-7$ atm. Its melting point (in the range of $144-150^{\circ}\mathrm{C}$) is higher than that of LDP (melting point $110^{\circ}\mathrm{C}$).

Question 84

What is osmotic pressure of solution of $1.7\ \mathrm{g}\ \mathrm{CaCl_2}$ in $1.25\ \mathrm{dm^3}$ water at $300\ \mathrm{K}$ if van't Hoff factor and molar mass of $\mathrm{CaCl_2}$, are 2.47 and $111\ \mathrm{g\ mol^{-1}}$ respectively?

$$\left[\mathrm{R}=0.082\ \mathrm{dm}^3\ \mathrm{atm\ mol}^{-1}\ \mathrm{K}^{-1}\right]$$

Options:

A. 0.625 atm

B. 0.744 atm

C. 0.827 atm

D. 0.936 atm

Answer: B

Solution:

$\pi=\mathrm{iMRT}=\frac{\mathrm{i} \times \mathrm{W}_2 \mathrm{RT}}{\mathrm{M}_2\ \mathrm{V}}$

$\pi=\frac{2.47 \times 1.7\ \mathrm{g} \times 0.082\ \mathrm{dm}^3\ \mathrm{atm\ mol}^{-1}\ \mathrm{K}^{-1} \times 300\ \mathrm{K}}{111\ \mathrm{g\ mol}^{-1} \times 1.25\ \mathrm{dm}^3}=0.744\ \mathrm{atm}$

Question 85

Which among the following pair of properties is intensive?

Options:

A. Mass and heat capacity

B. Heat capacity and pressure

C. Specific heat and pressure

D. Internal energy and boiling point

Answer: C

Solution:

A property which is independent of the amount of matter in a system is called intensive property. Examples: Specific heat and pressure

----

Question 86

Identify the molecular formula of an alkane that exhibits only two different structural isomers.

Options:

A. C_2H_6

B. C_3H_8

C. C_4H_{10}

D. C_6H_{12}

Answer: C

Solution:

Number of Carbon atoms	Alkane	Number of isomers
2	Ethane	No structural isomer

Number of Carbon atoms	Alkane	Number of isomers
3	Propane	No structural isomer
4	Butane	Two
6	Hexane	Five

Question 87

An organic compound 'A' on reaction with PCl_3 gives an alkyl chloride having formula C_3H_7Cl. 'A' on oxidation with PCC forms an aldehyde having formula C_3H_6O. Identify 'A'.

Options:

A. Propan-1-ol

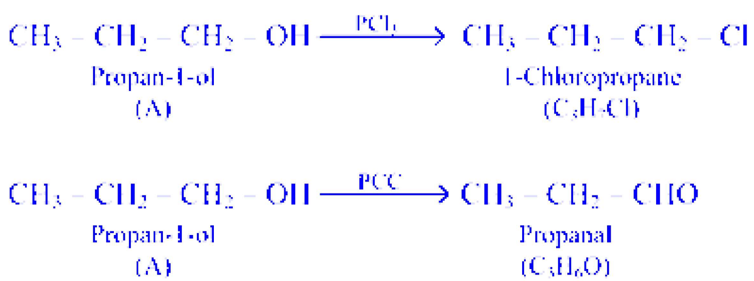
B. Propan-2-ol

C. Propanoic acid

D. Propanone

Answer: A

Solution:



Primary alcohols on oxidation form aldehydes. Hence, compound A should be primary alcohol. Therefore, only option (A) is valid.

Question 88

Identify tetrasaccharide from following.

Options:

A. Glycogen

B. Cellulose

C. Ribose

D. Stachyose

Answer: D

Solution:

(A)	Glycogen	Polysaccharide
(B)	Cellulose	Polysaccharide
(C)	Ribose	Monosaccharide
(D)	Stachyose	Tetrasaccharide

Question 89

What type of ligand does the ethylenediamine is?

Options:

A. Monodentate

B. Bidentate

C. Tetradentate

D. Hexadentate

Answer: B

Solution:

Ethylenediamine is a bidentate ligand. This means it has two donor atoms, which in the case of ethylenediamine, are the two nitrogen atoms. Each of these nitrogen atoms has a lone pair of electrons that can be donated to a central metal ion in a coordination complex. Therefore, the correct option is :

Option B : Bidentate

Question 90

What is angular momentum of an electron in fourth orbit of hydrogen atom?

Options:

A. $\frac{\mathrm{h}}{2 \pi}$

B. $\frac{\mathrm{h}}{\pi}$

C. $\frac{2 \mathrm{~h}}{\pi}$

D. $\frac{3 \mathrm{~h}}{\pi}$

Answer: C

Solution:

The angular momentum of an electron in a given stationary orbit of hydrogen atom

$=\mathrm{mvr}=\frac{\mathrm{nh}}{2 \pi} \quad(\text{where } \mathrm{n}=1,2,3)$

Angular momentum of an electron in fourth orbit of hydrogen atom $=\frac{4 h}{2 \pi}=\frac{2 h}{\pi}$

Question 91

Which from following complexes is heteroleptic?

Options:

A. $\mathrm{K}_3\left[\mathrm{Al}\left(\mathrm{C}_2 \mathrm{O}_4\right)_3\right]$

B. $\left[\mathrm{Co}\left(\mathrm{C}_2 \mathrm{O}_4\right)_3\right]^{3-}$

C. $\mathrm{Na}_3\left[\mathrm{AlF}_6\right]$

D. $\left[\mathrm{Co}\left(\mathrm{H}_2 \mathrm{O}\right)\left(\mathrm{NH}_3\right)_5\right]_3$

Answer: D

Solution:

Complexes in which metal ion is surrounded by more than one type of ligands are heteroleptic. Among the given complexes, option (D) is a heteroleptic complex.

Question 92

Calculate E_{cell}° in which following reaction occurs.

$$Mg_{(s)} + 2Ag^{+}_{(IM)} \rightarrow Mg^{+}_{(1M)} + 2Ag_{(s)} \text{ if } E^{\circ}_{Ag} = 0.8 \text{ V and } E^{\circ}_{Mg} = -2.37 \text{ V}$$

Options:

A. -3.17 V

B. 3.17 V

C. -1.57 V

D. 1.57 V

Answer: B

Solution:

For the given cell reaction, anode is Mg and cathode is Ag.

$$\begin{aligned} E^{o}_{\text{edl}} &= E^{\circ}_{\text{cuthode}} - E^{\circ}_{\text{unode}} \\ &= 0.8 - (-2.37) \\ &= 3.17 \text{ V} \end{aligned}$$

Question 93

Identify CORRECT decreasing order of solubilities of alcohols, alkanes and amines in water having comparable molar mass.

Options:

A. Alcohol > amine > alkane

B. Alkane > amine > alcohol

C. Amine $>$ alcohol $>$ alkane

D. Alkane $>$ alcohol $>$ amine

Answer: A

Solution:

Amines are less soluble in water as compared to alcohols having comparable molar mass, since $N-H$ bonds in amines are less polar than $O-H$ bond in alcohol.

Question 94

Which of the following characteristic properties is NOT true for crystalline solid?

Options:

A. These substances have sharp melting point,

B. These have different values of refractive index in different directions.

C. The constituent particles are orderly arranged.

D. These are isotropic.

Answer: D

Solution:

All crystalline substances except those having cubic structure are anisotropic.

Question 95

Time required for 90% completion of a first order reaction is 'x' minute. Calculate the time required to complete 99.9% of the reaction at same temperature.

Options:

A. x minute

B. $2x$ minute

C. $3x$ minute

D. $\frac{x}{2}$ minute

Answer: C

Solution:

$$\begin{aligned} t_{90\%6} &= \frac{2.303}{k} \log_{10} \frac{[A]_0}{[A]_t} = \frac{2.303}{k} \log_{10} \frac{100}{10} \\ &= \frac{2.303}{k} \log_{10} 10 \\ t_{99.9\%} &= \frac{2.303}{k} \log_{10} \frac{[A]_0}{[A]_t} = \frac{2.303}{k} \log_{10} \frac{100}{0.1} \\ &= \frac{2.303}{k} \log_{10} 1000 \\ \frac{t_{999\%}}{t_{90\%}} &= \frac{\frac{2.303}{k} \log_{10} 1000}{\frac{2.303}{k} \log_{10} 10} = \frac{\log_{10} 1000}{\log_{10} 10} = \frac{3}{1} \end{aligned}$$

$$\therefore \quad t_{99.9\%} = 3 \times t_{90\%} = 3x \text{ minute} \qquad (\text{since, } t_{90\%} = x \text{ minute})$$

Question 96

Identify 'A' in the following reaction:

$$\mathbf{A} \xrightarrow[\text{ether}]{\text{LiAlH}_4} \textbf{Ethanamine}$$

Options:

A. C_2H_5CN

B. CH_3CONH_2

C. $C_2H_5CONH_2$

D. CH_3NO_2

Answer: B

Solution:

$$CH_3CONH_2 \xrightarrow[\text{ether}]{LiAlH_4} \underset{\text{Ethanamine}}{CH_3CH_2NH_2}$$

Question 97

Which of the following reagents is used in Gatterman-Koch formylation of arene?

Options:

A. CrO_3

B. $CO, HCl/AlCl_{3\ (anhydr.)}$

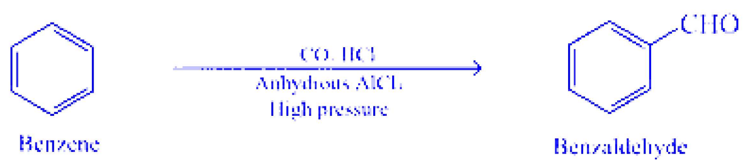
C. CrO_2Cl_2, CS_2

D. $Cl_2\ hv, H_3O^+$

Answer: B

Solution:

Gatterman-Koch formylation of arene involves treatment of benzene or substituted benzene under high pressure with carbon monoxide and hydrogen chloride gives benzaldehyde or substituted benzaldehyde. The reaction is carried out in presence of anhydrous aluminium chloride or cuprous chloride.



Question 98

Identify the last step in wet chemical synthesis of nanomaterial.

Options:

A. Formation of oxide or alcohol-bridged network

B. Dehydration

C. Aging of the gel

D. Drying of the gel

Answer: B

Question 99

Calculate the conductivity of 0.02 M electrolyte solution if its molar conductivity $407.2 \Omega^{-1} \mathrm{~cm}^2 \mathrm{~mol}^{-1}$?

Options:

A. $8.144 \times 10^{-3} \Omega^{-1} \mathrm{~cm}^{-1}$

B. $4.072 \times 10^{-3} \Omega^{-1} \mathrm{~cm}^{-1}$

C. $7.15 \times 10^{-3} \Omega^{-1} \mathrm{~cm}^{-1}$

D. $6.055 \times 10^{-3} \Omega^{-1} \mathrm{~cm}^{-1}$

Answer: A

Solution:

$$
\begin{aligned}
\Lambda_{\mathrm{m}} &= \frac{1000 \mathrm{k}}{\mathrm{c}} \\
\mathrm{k} &= \frac{\Lambda_{\mathrm{m}} \times \mathrm{c}}{1000} = \frac{407.2 \times 0.02}{1000} \\
&= 8.144 \times 10^{-3} \Omega^{-1} \mathrm{~cm}^{-1}
\end{aligned}
$$

Question 100

If 8.84 kJ heat is liberated for formation of 3 g ethane, calculate its $\triangle_{\mathrm{f}} \mathrm{H}^{\circ}$.

Options:

A. -8.00 kJ

B. 15.0 kJ

C. 30.0 kJ

D. $-84.4\ \mathrm{kJ}$

Answer: D

Solution:

Formation of ethane:

$2C_{(s)} + 3H_{2(g)} \longrightarrow C_2H_{6(g)}; \Delta H^\circ = ?$

Molar mass of $C_2H_6 = 30\ \mathrm{g\ mol}^{-1}$

$\therefore\quad 3\ \mathrm{g} C_2H_6 = \frac{3}{30} = \frac{1}{10}$ mol

$\therefore$ For formation of $\frac{1}{10}$ mol C_2H_6, 8.84 kJ heat is liberated.

$\therefore\quad$ Formation of 1 mol C_2H_6 will liberate 88.4 kJ of heat.

$\therefore\quad \Delta_f H^\circ$ of $C_2H_6 = -88.4\ \mathrm{kJ}$

Physics

Question 101

Two trains, each $30\ \mathrm{m}$ long are travelling in opposite directions with velocities $5\ \mathrm{m/s}$ and $10\ \mathrm{m/s}$. They will cross after

Options:

A. $4\ \mathrm{s}$

B. $3\ \mathrm{s}$

C. $2\ \mathrm{s}$

D. $1\ \mathrm{s}$

Answer: A

Solution:

Relative velocity of one train w. r. t other $= 5 + 0 = 15\ \mathrm{m/s}$

Total length to cross $(\mathrm{L}) = 30 + 30 = 60\ \mathrm{m}$

$\therefore \quad t = \frac{L}{V} = \frac{60}{15} = 4s$

Question 102

The fundamental frequency of air column in pipe 'A' closed at one end is in unison with second overtone of an air column in pipe 'B' open at both ends. The ratio of length of air column in pipe 'A' to that of air column in pipe ' B' is

Options:

A. 1 : 6

B. 3 : 8

C. 2 : 3

D. 3 : 4

Answer: A

Solution:

Fundamental frequency of a closed pipe, $n_1 = \frac{v}{4 L_1}$

Frequency of the second overtone, $n_2 = \frac{3v}{2L_2}$

Given $n_1 = n_2$

$\Rightarrow \frac{v}{4 L_1} = \frac{3v}{2 L_2}$

$\therefore \quad \frac{L_1}{L_2} = \frac{1}{6}$

Question 103

In Young's double slit experiment, the two slits are 'd' distance apart. Interference pattern is observed on a screen at a distance 'D' from the slits. A dark fringe is observed on a screen directly opposite to one of the slits. The wavelength of light is

Options:

A. $\frac{\mathrm{D}^2}{2\ \mathrm{d}}$

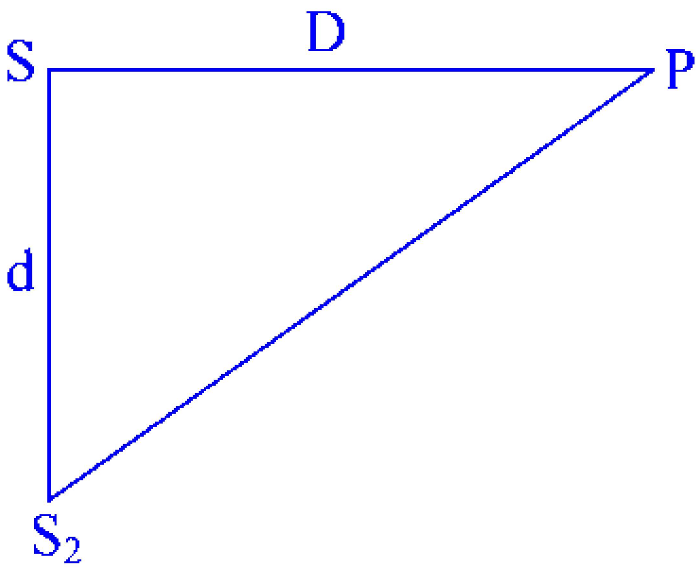
B. $\frac{\mathrm{d}^2}{2\mathrm{D}}$

C. $\frac{\mathrm{D}^2}{\mathrm{d}}$

D. $\frac{d^2}{D}$

Answer: D

Solution:



$$S_2P = (D^2 + d^2)^{1/2}$$
$$= D\left[1 + \frac{d^2}{D^2}\right]^{1/2}$$

Using binomial equation,

$\mathrm{S_2P} = \mathrm{D}\left[1 + \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{D}^2}\right]^{1/2} = \mathrm{D} + \frac{\mathrm{d}^2}{2\mathrm{D}}$

$\Rightarrow$ Path difference $= \frac{d^2}{2D}$

For dark fringe, $\frac{d^2}{2D} = \frac{\lambda}{2}$

$\therefore \quad \lambda = \frac{d^2}{D}$

Question 104

A black body radiates maximum energy at wavelength 'λ' and its emissive power is 'E' Now due to change in temperature of that body, it radiates maximum energy at wavelength $\frac{2\lambda}{3}$. At that temperature emissive power is

Options:

A. $\frac{81}{16}$

B. $\frac{27}{32}$

C. $\frac{18}{10}$

D. $\frac{9}{4}$

Answer: A

Solution:

From Stefan-Boltzmann's law,

$P = \frac{dQ}{dt} = A\sigma T^4$

Also, from Wien's displacement law,

$$\lambda_{max} = \frac{b}{T}(b \rightarrow \text{ Wien's constant })$$
$$\Rightarrow T = \frac{b}{\lambda}$$

$$\therefore \quad P = A \cdot \sigma\left(\frac{b}{\lambda}\right)^4$$
$$\Rightarrow P \propto \frac{1}{(\lambda)^4}$$

$\therefore$ Ratio of power dissipated is $\frac{P_2}{P_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^4$ Given $\lambda_1 = \lambda$ and $\lambda_2 = \frac{2\lambda}{3}$

$\therefore \quad \frac{P_2}{P_1} = \frac{(\lambda)^4}{\left(\frac{2\lambda}{3}\right)^4} = \frac{81}{16}$

Question 105

In meter bridge experiment, null point was obtained at a distance 'l' from left end. The values of resistances in the left and right gap are doubled and then interchanged. The new position of the null point is

Options:

A. $(100 - l)$

B. $(100 - 2l)$

C. $(100 - 3l)$

D. $\left(100 - \frac{l}{2}\right)$

Answer: A

Solution:

For a Metre bridge,

$\frac{X}{R} = \frac{l_X}{l_R}$

After doubling and interchanging the left and right gap resistances, the ratio of the resistances do not change.

This means the null point does not change.

$\therefore$ The new position will remain at $(100 - l)$.

Question 106

The magnetic field at the centre of a circular coil of radius 'R', carrying current $2A$ is 'B_1'. The magnetic field at the centre of another coil of radius '$3R$' carrying current $4A$ is 'B_2'. The ratio $B_1 : B_2$ is

Options:

A. 1 : 2

B. 2 : 1

C. 2 : 3

D. 3 : 2

Answer: D

Solution:

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi \times 2}{R} = \frac{\mu_0}{R}$$
$$B_2 = \frac{\mu_0}{4\pi} \times \frac{2\pi \times 4}{3R} = \frac{2\mu_0}{3R}$$
$$\therefore \quad \frac{B_1}{B_2} = \frac{\left(\frac{\mu_0}{R}\right)}{\frac{2\mu_0}{(3R)}} = \frac{3}{2}$$

Question 107

If a capacitor of capacity $900\ \mu F$ is charged to $100\ V$ and its total energy is transferred to a capacitor of capacity $100\ \mu F$, then its potential will be

Options:

A. 30 V

B. 200 V

C. 300 V

D. 400 V

Answer: C

Solution:

Let the unknown potential be V_u

Energy of a capacitor $= \frac{1}{2} CV^2$

$$\Rightarrow \frac{1}{2} C_1 V_1^2 = \frac{1}{2} C_2 (V_u)^2$$
$$\frac{1}{2} \times 900 \times 10^{-6} \times (100)^2 = \frac{1}{2} \times 100 \times 10^{-6} \times (V_u)^2$$
$$(V_u)^2 = 9 \times 100^2$$
$$\therefore \quad V_u = 300\ V$$

Question 108

The equation of wave is $Y = 6 \sin\left(12\pi t - 0.02\pi x + \frac{\pi}{3}\right)$ where 'x' is in m and 't' in s. The velocity of the wave is

Options:

A. $200 \mathrm{~m/s}$

B. $300 \mathrm{~m/s}$

C. $400 \mathrm{~m/s}$

D. $600 \mathrm{~m/s}$

Answer: D

Solution:

Given: $y = 6 \sin \left(12\pi t - 0.02\pi x + \frac{\pi}{3}\right)$

$\omega t = 12\pi t$

$\Rightarrow \frac{2\pi}{T} = 12\pi$

$\therefore \quad T = \frac{1}{6}$

$\Rightarrow n = 6$

Also given $\frac{2\pi}{\lambda} = 0.02\pi$

$\therefore \quad \lambda = \frac{2}{0.02} = 100$

From equation: $\mathrm{v} = \mathrm{n}\lambda$, we get

$v = 6 \times 100$

$= 600 \mathrm{~m/s}$

Question 109

With an alternating voltage source of frequency 'f', inductor 'L', capacitor 'C' and resistance 'R' are connected in series. The voltage leads the current by 45°. The value of 'L' is $(\tan 45^{\circ} = 1)$

Options:

A. $\left(\frac{1+2\pi \mathrm{fCR}}{4\pi^2 \mathrm{f}^2 \mathrm{C}}\right)$

B. $\left(\frac{1-2\pi \mathrm{fCR}}{4\pi^2 \mathrm{f}^2 \mathrm{C}}\right)$

C. $\left(\frac{4\pi^2 f^2 C}{1+2\pi f C R}\right)$

D. $\left(\frac{4\pi^2 f^2 C}{1-2\pi f C R}\right)$

Answer: A

Solution:

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\Rightarrow \tan 45^\circ = \left[\frac{2\pi f L - \frac{1}{2\pi f C}}{R}\right] \quad \ldots.(\because \phi = 45^\circ)$$

$$R = \frac{(2\pi f)^2 LC - 1}{2\pi f C}$$

$$\therefore \quad L = \frac{2\pi f CR + 1}{4\pi^2 f^2 C}$$

Question 110

A pure Si crystal has 4×10^{28} atoms per m^3. It is doped by 1 ppm concentration of antimony. The number of free electrons available will be

Options:

A. $4 \times 10^{34}\ m^{-3}$

B. $4 \times 10^{28}\ m^{-3}$

C. $4 \times 10^{22}\ m^{-3}$

D. $4 \times 10^{20}\ m^{-3}$

Answer: C

Solution:

Given: Density of Si atoms $= 4 \times 10^{28}$ atoms $/m^3$

After doping with 1 ppm of Sb,

No. of Sb atoms $= \frac{4 \times 10^{28}}{10^6}$

$= 4 \times 10^{22}$

The above number of Sb atoms donates 1 electron each.

$\therefore$ The total number of free electrons will be $4 \times 10^{22}\ \mathrm{m}^{-3}$

Question 111

A mass 'M' moving with velocity 'V' along X-axis collides and sticks to another mass 2M which is moving along Y-axis with velocity '3 V'. The velocity of the combination after collision is

Options:

A. $\frac{V}{3}\hat{\imath} + 2V\hat{\jmath}$

B. $\frac{V}{2}\hat{\imath} + V\hat{\jmath}$

C. $\frac{V}{3}\hat{\imath} - 2V\hat{\jmath}$

D. $\frac{V}{2}\hat{\imath} - V\hat{\jmath}$

Answer: A

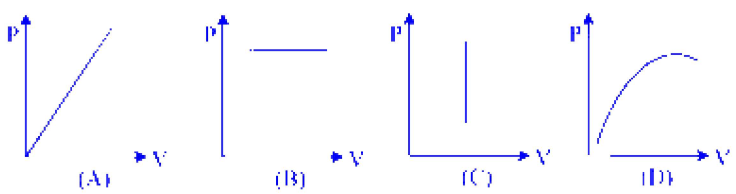
Solution:

From law of conservation of linear momentum,

$$M v \hat{i} + 2M(3v\hat{j}) = 3M\vec{v}$$
$$v\hat{i} + 6v\hat{j} = 3\vec{v}$$
$$\vec{v} = \frac{v}{3}\hat{\imath} + 2v\hat{\jmath}$$

Question 112

Which of the following graphs between pressure (P) and volume (V) correctly shows isochoric changes?



Options:

A. D

B. B

C. C

D. A

Answer: C

Solution:

Isochoric processes are those in which the volume of the system remains constant. In a Pressure-Volume (P-V) graph, an isochoric process is represented by a vertical line because the volume does not change, while the pressure can vary.

Looking at the provided graph :

- Graph A shows a diagonal line, indicating changes in both pressure and volume.
- Graph B shows a horizontal line, indicating constant pressure, not constant volume. This represents an isobaric process.
- Graph C shows a vertical line, which is characteristic of an isochoric process, where the volume remains constant while the pressure changes.
- Graph D shows a curve, indicating that both pressure and volume are changing.

Therefore, the correct option that represents an isochoric process is :

Option C : C

Question 113

A galvanometer has resistance 'G' and range 'Vg'. How much resistance is required to read voltage upto 'V' volt?

Options:

A. $G\left(\frac{V}{V_g}-1\right)$

B. $G\left(\frac{V+V_g}{V}\right)$

C. $G\left(\frac{V-V_g}{V}\right)$

D. GV_g

Answer: A

Solution:

Given: Resistance of the galvanometer = G

Range of the galvanometer $= V_g$

The series resistance value to be used for converting the galvanometer into a voltmeter of range 0 to $V_{g'}$ is,

$R = \frac{V_g}{I_g} - G$

Also,

$I_g = \frac{V_g}{G}$

To increase the measuring range to V, the new resistance value

$$R' = \frac{V}{\left(\frac{V_g}{G}\right)} - G$$
$$= \frac{VG}{V_g} - G = G\left(\frac{V}{V_g} - 1\right)$$

Question 114

'n' number of liquid drops each of radius 'r' coalesce to form a single drop of radius 'R'. The energy released in the process is converted into the kinetic energy of the big drop so formed. The speed of the big drop is

[T = surface tension of liquid, ρ = density of liquid.]

Options:

A. $\sqrt{\frac{T}{\rho}\left[\frac{1}{r} - \frac{1}{R}\right]}$

B. $\sqrt{\frac{2T}{\rho}\left[\frac{1}{r} - \frac{1}{R}\right]}$

C. $\sqrt{\frac{4T}{\rho}\left[\frac{1}{r} - \frac{1}{R}\right]}$

D. $\sqrt{\frac{6T}{\rho}\left[\frac{1}{r} - \frac{1}{R}\right]}$

Answer: D

Solution:

$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$ ..... (given)

$\therefore \quad R^3 = nr^3$

Energy released,

$\Delta U = T \times 4\pi r^2 \times n - T \times 4\pi R^2$

$= T \times 4\pi \frac{R^3}{r} - T \times 4\pi R^2$

This energy is converted into K.E

$$\therefore \quad \frac{1}{2}mv^2 = T \times 4\pi R^3 \left[\frac{1}{r} - \frac{1}{R}\right]$$
$$\Rightarrow \frac{1}{2}\rho \times \frac{4}{3}\pi R^3 \times v^2 = T \times 4\pi R^3 \left[\frac{1}{r} - \frac{1}{R}\right]$$
$$v^2 = \frac{6T}{\rho}\left[\frac{1}{r} - \frac{1}{R}\right]$$
$$\therefore \quad v = \sqrt{\frac{6T}{\rho}\left[\frac{1}{r} - \frac{1}{R}\right]}$$

Question 115

What is the moment of inertia of the electron moving in second Bohr orbit of hydrogen atom? [h = Planck's constant, m = mass of electron, ε_0 = permittivity of free space, e = charge on electron]

Options:

A. $\frac{4\varepsilon_0^2 h^4}{\pi^2 m e^4}$

B. $\frac{8m\varepsilon_0^2 h^4}{\pi^2 e^4}$

C. $\frac{16\varepsilon_0^2 h^4}{\pi^2 m e^4}$

D. $\frac{\varepsilon_0^2 h^4}{16\pi^2 \mathrm{me}^4}$

Answer: C

Solution:

Moment of Inertia $\mathrm{I} = \mathrm{MR}^2$

Radius of the n^{th} Bohr orbit is,

$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$

For $n = 2$,

$r_2 = \frac{4\varepsilon_0 h^2}{\pi m e^2}$

$\therefore$ Moment of inertia of the electron in the 2^{nd} orbit is

$$\begin{aligned} M.I &= m \times \left[\frac{4\varepsilon_0 h^2}{\pi m e^2}\right]^2 \\ &= m \times \frac{16\varepsilon_0^2 h^4}{\pi^2 m^2 e^4} \\ &= \frac{16\varepsilon_0^2 h^4}{\pi^2 m e^4} \end{aligned}$$

Question 116

At critical temperature, the surface tension of liquid is

Options:

A. zero

B. infinity

C. unity

D. same as that at any other temperature

Answer: A

Solution:

At the critical temperature of a liquid, the surface tension becomes zero. This happens because at the critical temperature, the properties of the liquid and gas phases become indistinguishable, leading to the disappearance of the liquid-gas interface. Surface tension is a property that arises due to the difference in intermolecular forces between the liquid and gas phases. When these phases become indistinguishable at the critical point, there's no longer a distinct interface, and thus no surface tension.

Therefore, the correct answer is :

Option A : zero

Question 117

Two wires $2 \mathrm{~mm}$ apart supply current to a $100 \mathrm{~V}, 1 \mathrm{~kW}$ heater. The force per metre between the wires is ($\mu_0=4 \pi \times 10^{-27}$ SI unit)

Options:

A. $2 \times 10^{-2} \mathrm{~N}$

B. $4 \times 10^{-3} \mathrm{~N}$

C. $2 \times 10^2 \mathrm{~N}$

D. $10^{-2} \mathrm{~N}$

Answer: D

Solution:

$$\text{Given: } P=1 \mathrm{~W}=1000 \mathrm{~W}, \ \mathrm{V}=100 \mathrm{~V}$$
$$\therefore \quad I=10 \mathrm{~A} \quad \ldots .(\because \mathrm{P}=\mathrm{VI})$$
$$\therefore \quad \mathrm{F}=\frac{\mu_0 \mathrm{I}_1 \mathrm{I}_2}{2 \pi \times \mathrm{a}}=\frac{4 \pi \times 10^{-7} \times 100}{2 \pi \times\left(2 \times 10^{-3}\right)}=10^{-7} \times 10^5=10^{-2} \mathrm{~N}$$

Question 118

A solenoid of 500 turns/m is carrying a current of 3 A. Its core is made of iron which has relative permeability 5001. The magnitude of magnetization is

Options:

A. $4.5 \times 10^6 \mathrm{~Am}^{-1}$

B. $6.0 \times 10^{-6} \mathrm{~Am}^{-1}$

C. $7.5 \times 10^6 \mathrm{~Am}^{-1}$

D. $9.0 \times 10^6 \mathrm{~Am}^{-1}$

Answer: C

Solution:

Given: $\mathrm{n} = 500$ turns$/\mathrm{m}, \mathrm{I} = 3$ A

$\mu_{\mathrm{r}} = 5001$
$\therefore \quad \mu = nI = 500 \times 3$ A
$= 1500$ A/m
But, $\chi_{\mathrm{m}} = \mu_{\mathrm{r}} - 1$
$= 5001 - 1$
$= 5000$
$\therefore \quad$ Magnetization $\mathrm{M} = \chi_{\mathrm{m}}\mathrm{H}$
$= 5000 \times 1500$
$= 7.5 \times 10^6 \mathrm{Am}^{-1}$

Question 119

The ratio of the velocity of the electron in the first Bohr orbit to that in the second Bohr orbit of hydrogen atom is

Options:

A. 8 : 1

B. 2 : 1

C. 4 : 1

D. 1 : 4

Answer: B

Solution:

Velocity of electron in the n^{th} orbit $\mathrm{V}_{\mathrm{n}} = \frac{\mathrm{Ze}^2}{2\varepsilon_0 \mathrm{nh}}$

$\Rightarrow \quad \mathrm{v}_{\mathrm{n}} \propto \frac{1}{\mathrm{n}}$
given $\mathrm{n}_1 = 1$ and $\mathrm{n}_2 = 2$,
$\therefore \quad \frac{\mathrm{v}_1}{\mathrm{v}_2} = \frac{\mathrm{n}_2}{\mathrm{n}_1} = \frac{2}{1}$

Question 120

The capacitive reactance of a capacitor 'C' is $X\Omega$. Both, the frequency of a.c. supply and capacitance of the above capacitor are doubled. The new capacitive reactance will be

Options:

A. $\frac{X}{4}\Omega$

B. $\frac{X}{2}\Omega$

C. $2X\Omega$

D. $4X\Omega$

Answer: A

Solution:

Given, $X_C = X\Omega$

$\Rightarrow \frac{1}{2\pi fC} = X\Omega$

New Capacitance $C = 2C$ and new frequency $f' = 2f$

$\therefore$ New capacitive reactance

$X'_C = \frac{1}{2\pi(2f)(2C)}$

$= \frac{1}{(2\pi)(4fC)}$

$= \frac{1}{4}X_C$

$= \frac{X}{4}\Omega$

Question 121

A $100\ \mathrm{mH}$ coil carries a current of $1\ \mathrm{A}$. Energy stored in the form of magnetic field is

Options:

A. 0.025 J

B. 0.050 J

C. 0.075 J

D. 0.100 J

Answer: B

Solution:

Energy stored in an inductor $E = \frac{1}{2} L I^2$

$$= \frac{1}{2} \times \left(100 \times 10^{-3}\right) \times 1$$
$$= 0.05 \text{ J}$$

Question 122

A metal rod 2 m long increases in length by 1.6 mm, when heated from 0°C to 60°C. The coefficient of linear expansion of metal rod is

Options:

A. $1.33 \times 10^{-5} /{}^{\circ}\text{C}$

B. $1.66 \times 10^{-5} /{}^{\circ}\text{C}$

C. $1.33 \times 10^{-3} /{}^{\circ}\text{C}$

D. $1.66 \times 10^{-3} /{}^{\circ}\text{C}$

Answer: A

Solution:

We know,

Coefficient of Linear expansion $\alpha = \frac{L_2 - L_1}{L_1 \Delta T}$ ...... (i)

Given: $\Delta T = 60 - 0 = 60^{\circ}\text{C}$

$L_1 = 2 \text{ m}$ and $L_2 = 2.0016$

Substituting the given values into (i),

$\alpha = \frac{.0016}{120} = 1.33 \times 10^{-5} /{}^{\circ}\text{C}$

Question 123

Assume that an electric field $\mathrm{E} = 30\mathrm{x}^2\hat{\mathrm{i}}$ exists in space. If 'V_0' is the potential at the origin and 'V_A' is the potential at $x = 2\ \mathrm{m}$, then the potential difference $(\mathrm{V}_\mathrm{A} - \mathrm{V}_0)$ is

Options:

A. -80 J

B. -120 J

C. 80 J

D. 120 J

Answer: A

Solution:

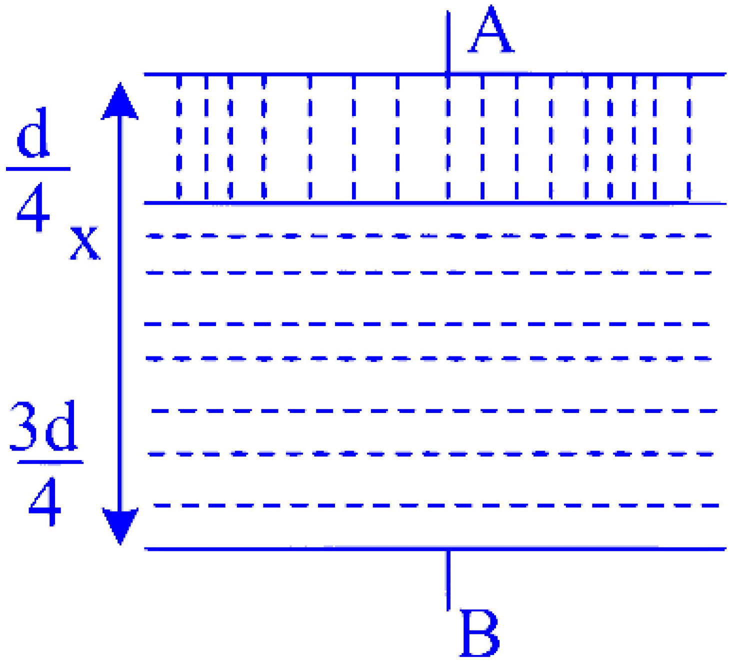
$$dV = \vec{E} \cdot dx$$

$$\int_{V_0}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_0 = -\left[10x^3\right]_0^2 = -80J$$

Question 124

Two dielectric slabs having dielectric constant 'K_1' and 'K_2' of thickness $\frac{\mathrm{d}}{4}$ and $\frac{3\ \mathrm{d}}{4}$ are inserted between the plates as shown in figure. The net capacitance between A and B is $[\varepsilon_0$ is permittivity of free space]



Options:

A. $\frac{2\ A\varepsilon_0}{d}\left[\frac{K_1\ K_2}{3\ K_1+K_2}\right]$

B. $\frac{3\ A\varepsilon_0}{d}\left[\frac{K_1+K_2}{K_1\ K_2}\right]$

C. $\frac{3\ A_0}{2\ d}\left[\frac{K_1+K_2}{K_1\ K_2}\right]$

D. $\frac{4A\varepsilon_0}{d}\left[\frac{K_1K_2}{3K_1+K_2}\right]$

Answer: D

Solution:

Capacity of 1^{st} Capacitor,

$C_1=\frac{K_1\varepsilon_0A}{d/4}=\frac{4K_1\varepsilon_0A}{d}$

Capacity of 2^{nd} Capacitor,

$C_2=\frac{K_2\varepsilon_0\ A}{3\ d/4}=\frac{4\ K_2\varepsilon_0\ A}{3\ d}$

Equivalent capacitance $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$

$$\frac{1}{C_1}=\frac{d}{4K_1\varepsilon_0 A};\frac{1}{C_2}=\frac{3d}{4K_2\varepsilon_0 A}$$

$$\frac{1}{C}=\frac{d}{4K_1\varepsilon_0 A}+\frac{3d}{4K_2\varepsilon_0 A}$$

$$\frac{1}{C}=\frac{d}{4\varepsilon_0 A}\left[\frac{1}{K_1}+\frac{3}{K_2}\right]$$

$$\frac{1}{C}=\frac{d}{4\varepsilon_0 A}\left[\frac{K_2+3K_1}{K_1K_2}\right]$$

$$\therefore \quad C=\frac{4\varepsilon_0 A}{d}\left[\frac{K_1K_2}{3K_1+K_2}\right]$$

Question 125

The output of an 'OR' gate is connected to both the inputs of a 'NAND' gate. The combination will serve as

Options:

A. OR gate

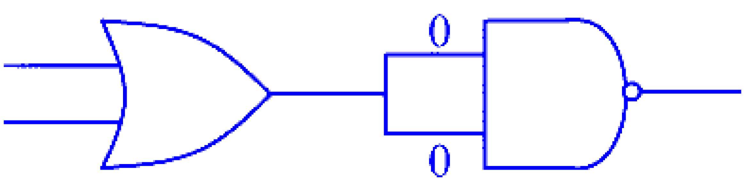
B. NOT gate

C. NOR gate

D. AND gate

Answer: C

Solution:



Output of OR gate	Output of NAND gate
0	1
1	0
1	0
1	0

Truth table of a NOR gate

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

The output matches with that of a NOR gate.

$\therefore$ The combination represents a NOR gate.

Question 126

Which one of the operations of $\mathrm{n-p-n}$ transistor differs from that of $p-n-p$ transistor?

Options:

A. The emitter-base junction is reversed biased in $\mathrm{n-p-n}$ transistor.

B. The emitter injects minority carriers into the base region of the $\mathrm{p-n-p}$ transistor.

C. The emitter injects holes into the base of the $\mathrm{p-n-p}$ transistor and electrons into the base region of $n-p-n$ transistor.

D. The emitter injects holes into the base of $\mathrm{n-p-n}$ transistor.

Answer: C

Question 127

A hollow metal pipe is held vertically and bar magnet is dropped through it with its length along the axis of the pipe. The acceleration of the falling magnet is ($\mathrm{g}=$ acceleration due to gravity)

Options:

A. equal to g.

B. less than g.

C. more than g.

D. zero.

Answer: B

Solution:

As a bar magnet falls through the pipe, an emf is induced in the body of the pipe due to change in magnetic flux. However, the free fall of the magnet will be opposed due to the development of eddy currents in accordance with Lenz's law. This process continues till the bar magnet attains a constant terminal velocity.

Question 128

A metal sphere of mass 'm' and density 'σ_1' falls with terminal velocity through a container containing liquid. The density of liquid is 'σ_2'. The viscous force acting on the sphere is

Options:

A. $mg\left(1+\frac{\sigma_2}{\sigma_1}\right)$

B. $mg\left(1-\frac{\sigma_1}{\sigma_2}\right)$

C. $\mathrm{mg}\left(1-\frac{\sigma_2}{\sigma_1}\right)$

D. $mg\left(1+\frac{\sigma_1}{\sigma_2}\right)$

Answer: C

Solution:

Given: Mass of sphere $= m$, Density of sphere $= \sigma_1$, Density of liquid $= \sigma_2$.

At $= \mathrm{v} = \mathrm{v_t}$,

Weight of sphere (W) = Viscous Force $(\mathrm{F_V})$ + Buoyant Force due to the medium $(\mathrm{F_B})$

$\Rightarrow \mathrm{W} = \mathrm{F_V} + \mathrm{F_B}$
$\mathrm{Mg} = \mathrm{F_V} + (\sigma_2\ \mathrm{V})\mathrm{g} \quad \ldots.(\because \mathrm{m} = \mathrm{D.V})$

$$\begin{aligned} \therefore \quad F_V &= mg - (\sigma_2 \mathrm{~V}) g \\ &= mg\left[1 - \frac{\sigma_2 \mathrm{~V}}{\mathrm{m}}\right] \\ &= mg\left[1 - \frac{\sigma_2 \mathrm{~V}}{\sigma_1 \mathrm{~V}}\right] \\ &= mg\left[1 - \frac{\sigma_2}{\sigma_1}\right] \end{aligned}$$

Question 129

Two uniform wires of same material are vibrating under the same tension. If the first overtone of first wire is equal to the 2^{nd} overtone of 2^{nd} wire and radius of the first wire is twice the radius of the 2^{nd} wire then the ratio of length of first wire to 2^{nd} wire is

Options:

A. 1 : 3

B. 3 : 1

C. 1 : 9

D. 9 : 1

Answer: A

Solution:

Fundamental frequency of the first wire is

$$\mathrm{n} = \frac{1}{2l_1}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}} = \frac{1}{2l_1}\sqrt{\frac{\mathrm{T}}{\pi \mathrm{r}_1^2 \rho}} = \frac{1}{2l_1 \mathrm{r}_1}\sqrt{\frac{\mathrm{T}}{\pi \rho}}$$

The first overtone $n_1 = 2n = \frac{1}{l_1 r_1}\sqrt{\frac{T}{\pi \rho}}$

Similarly, the second overtone of the second wire will be,

$$\mathrm{n}_2 = \frac{3}{2l_2 \mathrm{r}_2}\sqrt{\frac{\mathrm{T}}{\pi \rho}}$$

Given that $\mathrm{n}_1 = \mathrm{n}_2$

$$\therefore \quad \frac{1}{l_1 r_1}\sqrt{\frac{T}{\pi\rho}} = \frac{3}{2l_2 r_2}\sqrt{\frac{T}{\pi\rho}}$$

$$\therefore \quad 3l_1 r_1 = 2l_2 r_2$$

$$\frac{l_1}{l_2} = \frac{2r_2}{3r_1}$$

$$= \frac{2r_2}{3(2r_2)} \quad \ldots (\because r_1 = 2r_2)$$

$$= \frac{1}{3}$$

Question 130

A body is projected vertically from earth's surface with velocity equal to half the escape velocity. The maximum height reached by the satellite is (R = radius of earth)

Options:

A. R

B. $\frac{R}{2}$

C. $\frac{R}{3}$

D. $\frac{R}{4}$

Answer: C

Solution:

Given: $v = \frac{v_e}{2}$

If body is projected with velocity $v\ (v < v_e)$ then height up to which it will rise, $h = \frac{R}{\left(\frac{v_e^2}{v^2} - 1\right)}$

$$\therefore \quad h = \frac{R}{\left(\frac{v_e}{v_e/2}\right)^2 - 1} = \frac{R}{4-1} = \frac{R}{3}$$

Question 131

A ball of mass 'm' is dropped from a height 's' on a horizontal platform fixed at the top of a vertical spring. The platform is depressed by a

distance 'h'. The spring constant is ($g =$ acceleration due to gravity)

Options:

A. $\frac{2mg(s-h)}{h^2}$

B. $\frac{2mg(s+h)}{h^2}$

C. $\frac{mg(s-h)}{h}$

D. $\frac{mg(s+h)}{h}$

Answer: B

Solution:

From the question, it can be understood that the total distance the ball falls is $(S+h)$

The spring is compressed through a length h

$\therefore$ Loss of P.E. by the ball $= mg(S+h)$

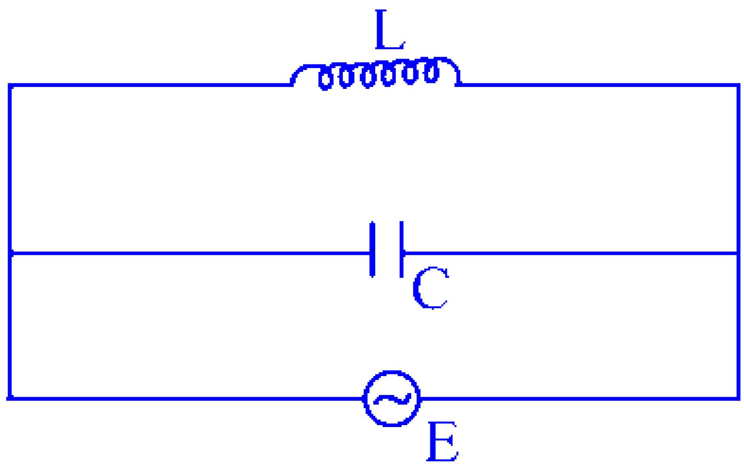
Work done on the spring $= \frac{1}{2}Kh^2$

Using law of conservation of energy,

$$\frac{1}{2}Kh^2 = mg(S+h)$$
$$\therefore \quad K = \frac{2mg(S+h)}{h^2}$$

Question 132

In the circuit given below, the current through inductor is 0.9 A and through the capacitor is 0.6 A. The current drawn from the a.c. source is



Options:

A. 1.5 A

B. 0.9 A

C. 0.6 A

D. 0.3 A

Answer: D

Solution:

As the currents in an inductor and capacitor are out of phase by 180°, we can write

Current through the capacitor $I_C = 0.9$ A

Current through the inductor $I_L = -(0.6 \text{ A})$

$$\therefore \quad \text{Total current drawn from the source } = I_C + I_L = 0.9 - 0.6 = 0.3 \text{ A}$$

Question 133

A body is executing a linear S.H.M. Its potential energies at the displacement 'x' and 'y' are 'E_1' and 'E_2' respectively. Its potential energy at displacement $(x + y)$ will be

Options:

A. $E_1 + E_2$

B. $\left(\sqrt{E_1}+\sqrt{E_2}\right)^2$

C. E_1-E_2

D. $\left(\sqrt{E_2}-\sqrt{E_1}\right)^2$

Answer: B

Solution:

We know,

Potential Energy $E_P=\frac{1}{2}Kx^2$

$\therefore \quad E_1=\frac{1}{2}Kx^2 \Rightarrow x=\sqrt{\frac{2E_1}{K}}$ .... (i)

and

$E_2=\frac{1}{2}Ky^2 \Rightarrow y=\sqrt{\frac{2E_2}{K}}$ ..... (ii)

Given, total displacement $=(x+y)$

$\therefore \quad$ Potential energy at displacement $(x+y)$,

E is $\frac{1}{2}$ $K(x+y)^2$

$$=\frac{1}{2}K\left(\sqrt{\frac{2E_1}{K}}+\sqrt{\frac{2E_2}{K}}\right)^2$$
$$=\frac{1}{2}K\left[\frac{2E_1}{K}+\frac{2E_2}{K}+2\left(\sqrt{\frac{E_1}{K}}\right)\left(\sqrt{\frac{2E_2}{K}}\right)\right]$$
$$=\left(\sqrt{E_1}+\sqrt{E_2}\right)^2=\left(E_1+E_2+2\sqrt{E_1E_2}\right)$$

Question 134

We have a jar filled with gas characterized by parameters P, V, T and another jar B filled with gas having parameters $2P, \frac{V}{4}, 2\,T$, where symbols have their usual meaning. The ratio of number of molecules in jar A to those in jar B is

Options:

A. 1 : 1

B. 1 : 2

C. 2 : 1

D. 4 : 1

Answer: D

Solution:

According to the gas equation, $\mathrm{PV} = \mathrm{Nk_B T}$

For the first gas, we have,

$\mathrm{PV} = \mathrm{N_1 k_B T}$ ..... (i)

For the second gas, we have,

$$(2\mathrm{P})\left(\frac{\mathrm{V}}{4}\right) = \mathrm{N_2 k_B}(2\,\mathrm{T})$$

$\mathrm{PV} = 4\,\mathrm{N_2 k_B T}$ .... (ii)

From equations (i) and (ii)

$\mathrm{N_1} = 4\,\mathrm{N_2} \Rightarrow \frac{\mathrm{N_1}}{\mathrm{N_2}} = 4$

Question 135

Two spheres each of mass 'M' and radius $\frac{R}{2}$ are connected at the ends of massless rod of length '$2R$'. What will be the moment of inertia of the system about an axis passing through centre of one of the spheres and perpendicular to the rod?

Options:

A. $\frac{2}{3}\mathrm{MR}^2$

B. $\frac{5}{2}\mathrm{MR}^2$

C. $\frac{5}{21}\mathrm{MR}^2$

D. $\frac{21}{5}\mathrm{MR}^2$

Answer: D

Solution:

From parallel axis theorem,

$I_o = I_c + Mh^2$

Let the moment of inertia of sphere 1 be

$I_1 = \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(2R)^2$

and,

Let the moment of inertia of sphere 2 be

$I_2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2$

Moment of inertia of the rod $I_3 = 0$

$\therefore$ Moment of inertia of the system,

$$I = I_1 + I_2 + I_3$$
$$I = \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(2R)^2 + \frac{2}{5}M\left(\frac{R}{2}\right)^2$$
$$= \frac{4}{5}M\left(\frac{R}{2}\right)^2 + 4MR^2$$
$$= \frac{1}{5}MR^2 + 4MR^2$$
$$= \frac{21}{5}MR^2$$

Question 136

If the potential difference used to accelerate electrons is doubled, by what factor does the de-Broglie wavelength associated with electrons change?

Options:

A. Wavelength in increased to $\frac{1}{\sqrt{2}}$ times.

B. Wavelength in increased to $\frac{1}{2}$ times.

C. Wavelength in decreased to $\frac{1}{\sqrt{2}}$ times.

D. Wavelength in decreased to $\frac{1}{2}$ times.

Answer: C

Solution:

From $\lambda = \dfrac{\mathrm{h}}{\sqrt{2\mathrm{mqV}}}$,

$\lambda \propto \dfrac{1}{\sqrt{\mathrm{V}}}$

If potential difference is doubled, $\lambda \propto \frac{1}{\sqrt{2\ \mathrm{V}}}$

$\therefore \quad \lambda$ is decreased by $\frac{1}{\sqrt{2}}$ times.

Question 137

A simple harmonic progressive wave is represented by $y = A\sin(100\pi t + 3x)$. The distance between two points on the wave at a phase difference of $\frac{\pi}{3}$ radian is

Options:

A. $\frac{\pi}{8}$ m

B. $\frac{\pi}{9}$ m

C. $\frac{\pi}{6}$ m

D. $\frac{\pi}{3}$ m

Answer: B

Solution:

Equation of the given harmonic progressive wave $y = A\sin(100\pi t + 3)$ ...... (i)

General equation of a harmonic wave

$\mathrm{y} = \mathrm{A}\sin(\mathrm{wt} + \mathrm{kx})$ ..... (ii)

From equations (i) and (ii), $\omega = 100\pi, k = 3$

But, $\mathrm{k} = \frac{2\pi}{\lambda} \Rightarrow 3 = \frac{2\pi}{\lambda}$

$\therefore \quad \lambda = \frac{2\pi}{3}$

We also know,

Path difference $\Delta x = \frac{\lambda}{2\pi} \times$ Phase difference $\Delta \phi$

$$\therefore \quad \Delta x = \frac{2\pi}{3 \times 2\pi} \times \frac{\pi}{3} \quad \ldots \left(\text{Given } \Delta \phi = \frac{\pi}{3} \right)$$
$$= \frac{\pi}{9} \text{ m}$$

Question 138

An insulated container contains a monoatomic gas of molar mass 'm'. The container is moving with velocity 'V'. If it is stopped suddenly, the change in temperature of a gas is [R is gas constant]

Options:

A. $\frac{\mathrm{MV}^2}{\mathrm{R}}$

B. $\frac{MV^2}{2R}$

C. $\frac{\mathrm{MV}^2}{3\mathrm{R}}$

D. $\frac{3\mathrm{MV}^2}{2\mathrm{R}}$

Answer: C

Solution:

Kinetic energy loss of the gas is

$\Delta \mathrm{E} = \frac{1}{2} MV^2 \cdot \mathrm{n}$ ..... (i) ....(where n is the no. of moles of the gas)

Heat gained by the gas due to temperature change ΔT is $\Delta Q = {}_n C_V \Delta T$

But, $\mathrm{C_V} = \frac{3}{2}\mathrm{R}$ ....(gas is monoatomic)

$\therefore \quad \Delta \mathrm{Q} = \frac{3}{2}\mathrm{R} \cdot \mathrm{n}\Delta \mathrm{T}$ ..... (ii)

Equating (i) and (ii),

$$\Delta \mathrm{E} = \Delta \mathrm{Q}$$
$$\frac{1}{2}\mathrm{MV}^2 \cdot \mathrm{n} = \frac{3}{2}\mathrm{R} \cdot \mathrm{n}\Delta \mathrm{T}$$
$$\Delta \mathrm{T} = \frac{\mathrm{MV}}{3\mathrm{R}}$$

Question 139

To get three images of a single object, the angle between the two plane mirrors should be

Options:

A. 50°

B. 60°

C. 72°

D. 90°

Answer: D

Solution:

As the object is placed symmetrically,

$n = \left(\frac{360^{\circ}}{\theta} - 1\right) \Rightarrow 3 = \left(\frac{360^{\circ}}{\theta} - 1\right) \Rightarrow \theta = 90^{\circ}$

Question 140

A parallel beam of monochromatic light falls normally on a single narrow slit. The angular width of the central maximum in the resulting diffraction pattern

Options:

A. increases with increase of slit width.

B. decreases with increase of slit width.

C. decreases with decrease of slit width.

D. may increase or decrease with decrease of slit width.

Answer: B

Solution:

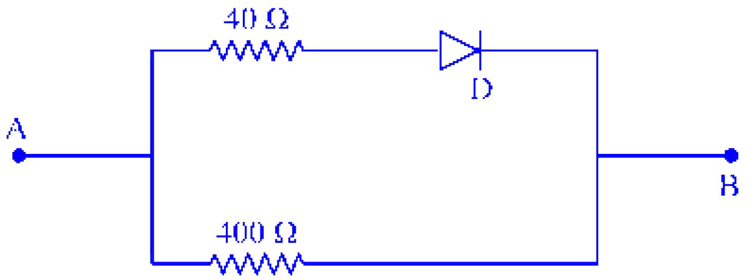
Width of central maximum $W_c = 2\left[\frac{\lambda D}{a}\right]$

$\Rightarrow W_c \propto \frac{1}{a}$

$\therefore$ Angular width of principal maximum decreases with increase in width of the slit.

Question 141

For the diagram shown, the resistances between points A and B when ideal diode D is forward biased is 'R_1' and that when reverse biased is 'R_2'. The ratio $R_1 : R_2$ is



Options:

A. 2 : 1

B. 1 : 1

C. 1 : 2

D. 1 : 4

Answer: C

Solution:

When the diode is forward biased, current will flow through both the arms.

$\therefore$ The effective resistance is

$R_1 = \frac{40 \times 40}{80} = \frac{1600}{80} = 20\Omega$

When the diode is reverse biased, current will flow through the bottom arm only

$\therefore$ The effective resistance R_2 is 40Ω.

$\therefore \frac{R_1}{R_2} = \frac{20}{40} = \frac{1}{2}$

Question 142

The maximum kinetic energy of the photoelectrons varies

Options:

A. inversely with the intensity of incident radiation and is independent of its frequency.

B. inversely with the frequency of incident radiation and is independent on its intensity.

C. linearly with the frequency of incident radiation and depends on its intensity

D. linearly with the frequency of incident radiation and is independent of its intensity.

Answer: D

Question 143

Light waves from two coherent sources arrive at two points on a screen with path difference of zero and $\frac{\lambda'}{2}$. The ratio of intensities at the points is $(\cos 0^\circ = 1, \cos \pi = -1)$

Options:

A. $2:1$

B. $1:1$

C. $1:2$

D. $\infty:1$

Answer: D

Solution:

Given: Wave length $= \frac{\lambda}{2}$

Path difference of first wave $\Delta x_1 = 0$

Path difference of second wave $\Delta x_2 = \frac{\lambda}{2}$

$$\therefore \quad \Delta\phi_1 = \frac{2\pi}{\lambda} \cdot \Delta x_1 = 0$$

Similarly,

$$\Delta\phi_2 = \frac{2\pi}{\lambda} \cdot \Delta x_2 = \pi$$

$\therefore$ Intensity of first wave $\mathrm{I}_1 = 4\mathrm{I}_0 \cos^2(0) = 4\mathrm{I}_0$

Similarly,

Intensity of second wave $I_2 = 4I_0 \cos^2\left(\frac{\pi}{2}\right) = 0$

$$\therefore \quad \frac{\mathrm{I}_1}{\mathrm{I}_2} = \frac{4\mathrm{I}_0}{0} = \infty$$

$$\Rightarrow \infty : 1$$

Question 144

A uniform rope of length 'L' and mass 'm_1' hangs vertically from a rigid support. A block of mass 'm_2' is attached to the free end of the rope. A transverse wave of wavelength 'λ_1' is produced at the lower end of the rope. The wavelength of the wave when it reaches the top of the rope is 'λ_2'. The ratio $\frac{\lambda_1}{\lambda_2}$ is

Options:

A. $\left[\frac{m_2}{m_1+m_2}\right]^{\frac{1}{2}}$

B. $\left[\frac{m_1+m_2}{m_2}\right]^{\frac{1}{2}}$

C. $\left[\frac{\mathrm{m}_1}{\mathrm{m}_1+\mathrm{m}_2}\right]^{\frac{1}{2}}$

D. $\left[\frac{\mathrm{m}_2}{\mathrm{m}_1-\mathrm{m}_2}\right]^{\frac{1}{2}}$

Answer: A

Solution:

Let velocity of pulse at lower end be v_1 and at top be v_2

$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{\mathrm{v}_2}{\mathrm{v}_1} \quad \left(\because \lambda = \frac{\mathrm{v}}{\mathrm{n}} \text{ and n} = \text{ constant}\right)$

Velocity of transverse wave on a string is

$$v=\sqrt{\frac{T}{m}}$$

where, m is linear density.

In this case, $v \propto \sqrt{T}$

$$\therefore \quad \frac{\lambda_2}{\lambda_1}=\frac{v_2}{v_1}=\sqrt{\frac{T_2}{T_1}}=\sqrt{\frac{(m_2+m_1)}{m_2}}$$

Where, T_2 is tension at upper end of rope and T_1 is tension at lower end of rope.

$$\Rightarrow \frac{\lambda_1}{\lambda_2}=\sqrt{\frac{m_2}{m_2+m_1}}$$

Question 145

In a vessel, the ideal gas is at a pressure P. If the mass of all the molecules is halved and their speed is doubled, then resultant pressure of the gas will be

Options:

A. 4P

B. 2P

C. P

D. $\frac{P}{2}$

Answer: B

Solution:

We know,

$v_{ms}^2 = \frac{3PV}{Nm}$

$\Rightarrow P = \frac{1}{3}\frac{mN}{V} v_{mms}^2$

$\Rightarrow P \propto v_{ms}^2$

$\therefore \quad \frac{P_2}{P_1} = \frac{m_2}{m_1} \times \left[\frac{v_2}{v_1}\right]^2$

$= \frac{\left(\frac{m_1}{2}\right)}{m_1}\left[\frac{2v_1}{v_1}\right]^2$

$\ldots\ldots\left(\because \text{ given } m_2 = \frac{m_1}{2} \text{ and } v_2 = 2v_1\right)$

$\frac{P_2}{P_1} = 2$

$\therefore \quad P_2 = 2P_1$

$= 2P \quad \ldots.(\text{given } P_1 = P)$

Question 146

Two concentric circular coils having radii r_1 and r_2 $(r_2 << r_1)$ are placed co-axially with centres coinciding. The mutual induction of the arrangement is (Both coils have single turn, $\mu_0 =$ permeability of free space)

Options:

A. $\frac{\mu_0 \pi r_2^2}{2r_1}$

B. $\frac{\mu_0 \pi r_2}{2r_1}$

C. $\frac{\mu_0 \pi r_2^2}{r_1^2}$

D. $\frac{\mu_0 \pi r_2}{r_1}$

Answer: A

Solution:

Let I_1 be the current through the coil whose radius is r_1.

$\therefore$ Magnetic field at the centre of the coil,

$B_1 = \frac{\mu_0 I_1}{2r_1}$

Magnetic flux passing through the coil of radius r_2 is

$$\phi_2 = B_1 \cdot \pi r_2^2 \quad (\because \phi = B.A)$$
$$= \frac{\mu_0 I_1}{2r_1} \cdot \pi r_2^2$$

$\therefore$ Mutual inductance of the arrangement,

$$M = \frac{\phi_2}{I_1}$$
$$= \frac{\mu_0 I_1 \pi r_2^2}{2r_1 I_1}$$
$$= \frac{\mu_0 \pi r_2^2}{2r_1}$$

Question 147

A system consists of three particles each of mass 'm_1' placed at the corners of an equilateral triangle of side '$\frac{L}{3}$', A particle of mass 'm_2' is placed at the mid point of any one side of the triangle. Due to the system of particles, the force acting on m_2 is

Options:

A. $\frac{3Gm_1 m_2}{L^2}$

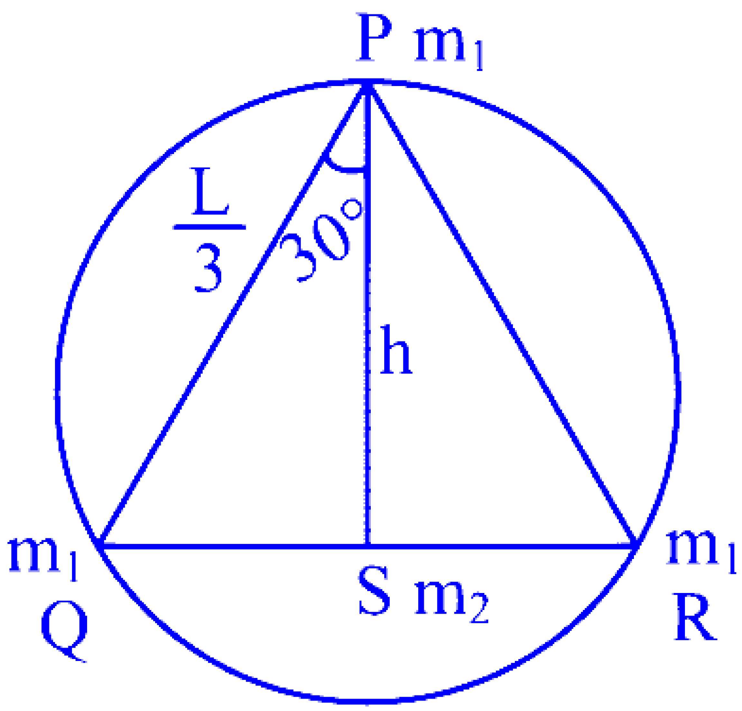
B. $\frac{6Gm_1 m_2}{L^2}$

C. $\frac{9Gm_1 m_2}{L^2}$

D. $\frac{12Gm_1 m_2}{L^2}$

Answer: D

Solution:



From the fig., we can see that the forces due to masses at Q and R cancel each other as they are equal and opposite. The force at P is only due to m_1.

In $\triangle PQS$,

$h = \frac{L}{3} \cos 30^\circ = \frac{L\sqrt{3}}{6}$

$\therefore$ Force on m_2 due to m_1 at P is

$$F = \frac{Gm_1m_2}{\left(\frac{L\sqrt{3}}{6}\right)^2} = \frac{Gm_1m_2 \cdot 12}{L^2} = \frac{12Gm_1\,m_2}{L^2}$$

Question 148

The moment of inertia of a uniform square plate about an axis perpendicular to its plane and passing through the centre is $\frac{Ma^2}{6}$, where 'M' is the mass and 'a' is the side of square plate. Moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

Options:

A. $\frac{Ma^2}{6}$

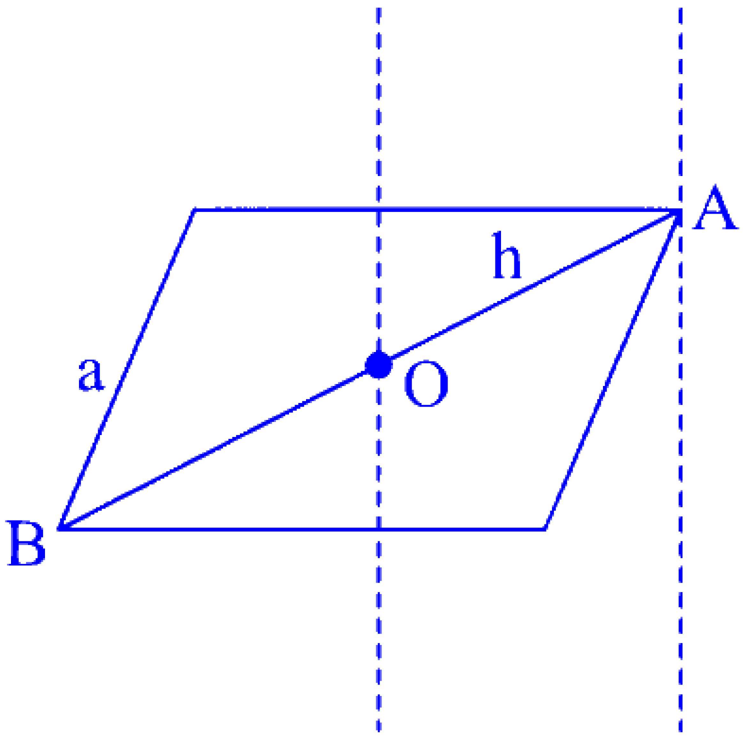
B. $\frac{2Ma^2}{3}$

C. $\frac{Ma^2}{3}$

D. $\frac{2Ma^2}{5}$

Answer: B

Solution:



$$I_O = \frac{Ma^2}{6}$$
$$AB = \sqrt{2a^2} = \sqrt{2}a$$
$$\therefore \quad AO = \frac{a}{\sqrt{2}} = h$$
$$\therefore \quad I_A = I_0 + Mh^2$$
$$= \frac{Ma^2}{6} + \frac{Ma^2}{2}$$
$$= \frac{8Ma^2}{12} = \frac{2}{3}Ma^2$$

Question 149

Two lenses of power $-15\mathrm{D}$ and $+5\mathrm{D}$ are in contact with each other. The focal length of the combination is

Options:

A. $-0.1\ \mathrm{cm}$

B. $-10\ \mathrm{cm}$

C. $-20\ \mathrm{cm}$

D. $+10\ \mathrm{cm}$

Answer: B

Solution:

$P=\frac{1}{f}$

For combination of lenses,

$\mathrm{P}=\mathrm{P}_1+\mathrm{P}_2$

Given: $P_1=-15D$ and $P_2=+5F$

$$\therefore \quad P=-10D$$
$$\Rightarrow f=-\frac{1}{10}m$$
$$\therefore \quad f=-10\ \mathrm{cm}$$

Question 150

A particle performing uniform circular motion of radius $\frac{\pi}{2}$ m makes 'x' revolutions in time 't'. Its tangential velocity is

Options:

A. $\frac{\pi \mathrm{x}}{\mathrm{t}}$

B. $\frac{\pi x^2}{t}$

C. $\frac{\pi^2 x^2}{t}$

D. $\frac{\pi^2 \mathrm{x}}{\mathrm{t}}$

Answer: D

Solution:

$$\text{Circumference of the circle } = 2\pi r$$
$$= 2\pi \times \frac{\pi}{2} = \pi^2$$
$$\therefore \quad \text{Tangential velocity } = \frac{\text{Distance Travelled}}{\text{Time}}$$
$$= \frac{\pi^2 \times x}{t}$$
$$= \frac{\pi^2 x}{t}$$
